# A Three-Conductor Elementary Clogston Coaxial Transmission Line—Calculation, Fabrication and Experiment

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*Eddy current losses in a conductor of a transmission line can be reduced if the conductor can be divided into two parallel shells and the current divided evenly between these two. According to the Clogston theory, this may be achieved if the dielectric constant of the space between these two shells is less than that of the lines' main dielectric by the right amount.*

*This paper describes the calculation of the properties of a three-conductor Clogston line, its fabrication, the measurement of its attenuation, and additional calculations relating to the termination of the line.*

*The calculations indicate that with a line having this structure attenuation can be reduced 24 per cent at 4 mc and about 15 per cent at 1 mc and 10 mc from that of a conventional coaxial line having the same dimensions as the one studied here. A reduction of 18 per cent instead of 24 per cent was realized in the 600-foot length of line which was built. About one-half of this difference is a result of the ideal ratio of dielectric constants not having been attained in this first attempt.*

*It was found that ideal termination of the line is difficult. However, a simple approximation to an ideal termination yields an insertion loss for the line only a small amount larger than the ideal, provided the length of line is such that its loss is about 20 db.*

## I. INTRODUCTION

A three-conductor coaxial cable, in which the locations of the two inner conductors are transposed at regular intervals, was built and shown to have about 20 per cent lower attenuation over a certain frequency interval than a two-conductor coaxial of the same size.[1] This is because the eddy current losses are smaller, and this in turn is a result of the normal central conductor having been divided into two and the normal central current having been divided nearly evenly between these two.

It is possible to do this so that the two conductors occupy approximately the same space as the single one did before, since the skin effect causes most of the current to flow in a thin shell at the outer periphery of the inner conductor over a certain frequency interval. Dividing the central conductor provides two shells for current flow.

One way to achieve equal currents in the two inner shells is to transpose the two inner conductors frequently enough in the manner of litz wire and so force the even division. This is the method used in the project described in Ref. 1. Another method is to so choose the dimensions and properties of the two dielectric spaces that the line has a natural mode of propagation with approximately the desired current distribution. This is a special case of the method suggested by Clogston a number of years ago,[2] namely, that the conductors of a transmission line be made of many laminations with proper dielectric separation. Other work has also been done on the three-conductor case.[3] It is also possible to obtain the desired current distribution by having the inductances associated with the several spaces in the proper ratio. This has been done in experimental lines built in Japan.[4] The proper values of inductance are achieved by having spiral gaps of increasing pitch as the center of the cable is approached.

The Clogston method which is used in the experiment described in this paper depends on having the proper ratio of dielectric constants in the several spaces. Fig. 1 is a pictorial representation of the Clogston line built for our experiment. This line is uniform with respect to length, in contrast with the transposed line. Fig. 2 is an enlarged photograph of the central member of the cable.

Following Clogston's work on laminated conductors for transmission lines,[2] an experimental line having 100 laminations was built.[5] Measurements on this cable did not show the expected reduction of attenuation.[5]

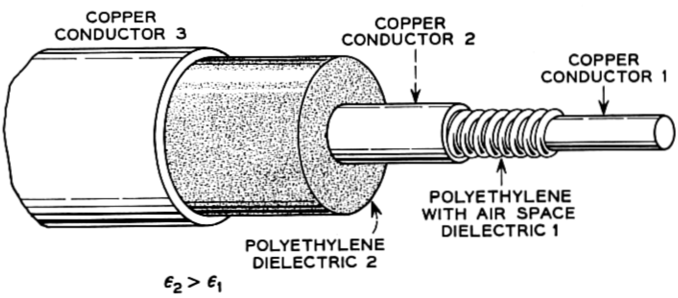


Fig. 1 — Drawing showing construction of three-conductor Clogston transmission line.

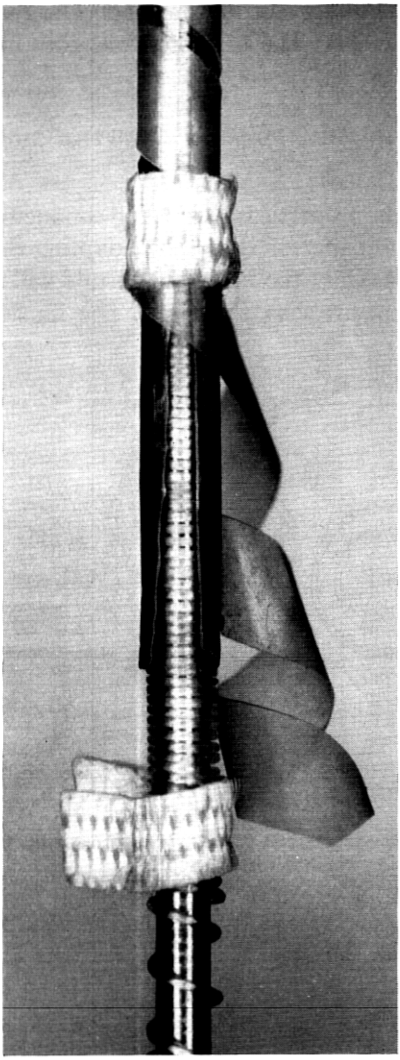

Fig. 2 — Enlarged photograph of two-conductor central member of Clogston line.

It was suspected that nonuniformity in spacing and thickness of laminations had a part in the discrepancy, and some calculations of this were made.[6] But the discrepancy was not resolved until the work[7] by Gordon Raisbeck was carried out. Meanwhile, it was decided to build the simplest laminated line, namely one in which the central member consisted of two conductors instead of one. The first result of this program was the transposed line.[1] The next step was the three-conductor Clogston line project reported in the present paper.

## II. CALCULATION OF THE PROPERTIES OF THE THREE-CONDUCTOR COAXIAL LINE

The transmission properties of a three-conductor coaxial structure were derived in the paper of Ref. 1 by adapting Schelkunoff's general results[8] to this case. Only the necessary results will be repeated here.

Sections of the line are shown in Fig. 3. This illustration and the nota-

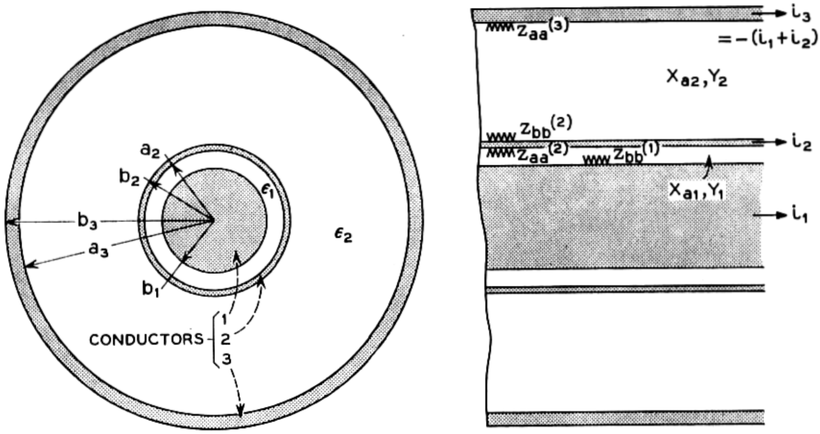


Fig. 3 — The three-conductor Clogston line: cross section and schematic longitudinal section showing surface impedances.

tion used here are the same as in Ref. 1. The conductors are numbered 1, 2, 3 beginning at the innermost. The currents flowing in these conductors and the voltages on them are designated $i_1$, $i_2$, $i_3$; $v_1$, $v_2$, $v_3$ in same order. It is seen that

$$-i_3 = i_1 + i_2 \quad \text{and}$$

$$v_3 = 0.$$

The behavior of the line is described by the two homogeneous equations

$$\begin{aligned} [Z_{11} - \gamma^2/Y_1]i_1 - Z_{ab}(i_1 + i_2) &= 0 \\ -Z_{ab}i_1 + [Z_{22} - \gamma^2/Y_2](i_1 + i_2) &= 0 \end{aligned} \tag{1}$$

and two others

$$\begin{aligned} v_2 - v_1 &= -\gamma i_1/Y_1 \\ v_2 &= \gamma(i_1 + i_2)/Y_2 \end{aligned} \tag{2}$$

where

$$Z_{11} = Z_{bb}^{(1)} + Z_{aa}^{(2)} + jX_{a_1} \tag{3}$$

is the effective series impedance of the simple coaxial line consisting of conductors 1 and 2 only and where

$$Z_{22} = Z_{bb}^{(2)} + Z_{aa}^{(3)} + jX_{a_2} \tag{4}$$

is the effective series impedance of the simple coaxial line consisting of conductors 2 and 3 only. In these expressions $Z_{bb}^{(m)}$ and $Z_{aa}^{(m)}$ are the outer and inner surface impedances, respectively, of conductor $m$, using Schelkunoff's concept and notation described in Ref. 8. The surface impedances are fictitious coefficients which gather the impedance effects of a conductor at its surfaces. $Z_{ab}^{(2)}$ is the surface *transfer* impedance of conductor 2; $\gamma$ is the propagation constant. Also

$$jX_{a_1} = j\mu \frac{\omega}{2\pi} \log (a_2/b_1)$$

$$Y_1 = j \frac{2\pi\omega\epsilon_1}{\log (a_2/b_1)}$$

and

$$jX_{a_2} = j\mu \frac{\omega}{2\pi} \log (a_3/b_2)$$

$$Y_2 = j \frac{2\pi\omega\epsilon_2}{\log (a_3/b_2)} \tag{5}$$

are the series impedances and shunt admittances of the two dielectric spaces between conductors 1 and 2 and between conductors 2 and 3, respectively. All these coefficients are per unit length of line.

The propagation of voltages and currents in the natural modes of the line are governed by the four values of $\gamma$ which alone permit the two homogeneous equations (1) to be satisfied. These values of $\gamma$ are roots of the characteristic equation

$$\gamma^4 - \gamma^2(Y_1Z_{11} + Y_2Z_{22}) + Y_1Y_2(Z_{11}Z_{22} - Z_{ab}^2) = 0 \tag{6}$$

obtained by setting the determinant of coefficients of the homogeneous equations equal to zero. The roots occur in pairs, so that we have

$$\begin{aligned} \gamma_1 &= -\gamma_2 \\ \gamma_3 &= -\gamma_4 . \end{aligned} \tag{7}$$

For the dimensions considered here, $\gamma_1$ and $\gamma_3$ are associated with low- and high-loss modes respectively.

If it is assumed a priori that the currents $i_1$ and $i_2$ in the two laminations are equal, and if the dimensions and impedances of the three-conductor cable used for the transposed line are taken, then it is found that attenuation of the low-loss mode is reduced about 30 per cent below that of a comparable two-conductor coaxial. However, these assumptions require the ratio of dielectric constants to be complex, viz.

$$\epsilon_2/\epsilon_1 = 1.23 - j\,0.03 \tag{8}$$

which implies dissipation in at least one of the spaces. This would introduce additional loss which has not been considered.

Instead of doing further analytical work along these lines, we resorted to numerical computation of the line properties, using again the dimensions of the uniform three-conductor cable made for the transposed line (OD about 0.18 inch), but with real values for the ratio $\epsilon_1/\epsilon_2$. The values of attenuation constant versus ratio $\epsilon_1/\epsilon_2$ of dielectric constants for several values of spacing between conductors 1 and 2 found in these calculations are plotted in Fig. 4. With these real values for the ratio $\epsilon_1/\epsilon_2$, there is an appreciable (about 24 per cent) reduction of attenuation at the minimum. Near this minimum, the ratio of currents in conductors 1 and 2 is nearer 1.2, as may be seen in Fig. 5, where the ratio of voltages is plotted also. It will be noticed in Fig. 4 that the curve of attenuation versus $\epsilon_1/\epsilon_2$ is considerably sharper where the lamination spacing is 15 mils instead of 5 mils. In Fig. 6 the attenuation constants $\alpha_1$ and $\alpha_3$ of the low- and high-loss modes, respectively, and also that of a reference two-conductor coaxial, are plotted versus frequency. This reference coaxial has the same size outer conductor as that of the special cable and a solid inner conductor with an OD very nearly the same as that of conductor 2; the dielectric is specified by $\epsilon_2$. The shape of the $\alpha_1$ curve is similar to that for the transposed line, except that attenuation at low frequencies approaches that of the reference two-conductor coaxial, whereas attenuation in the transposed line was somewhat higher. The phase distortion of the three-conductor Clogston and the two-conductor reference line are plotted in Fig. 7. For further comparison of the Clogston line, attenuation and phase distortion of the two-conductor

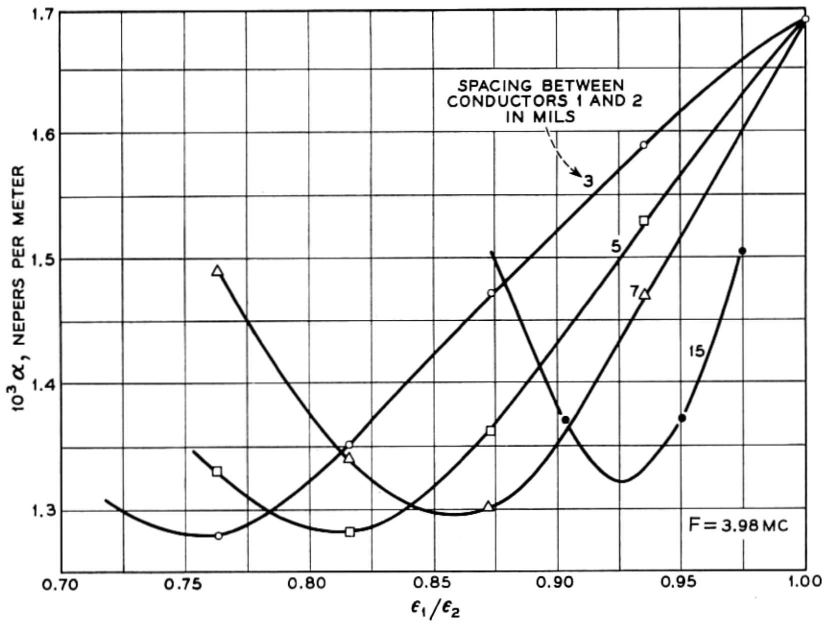


Fig. 4 — Calculated attenuation of three-conductor Clogston line.

coaxial consisting of conductors 2 and 3 — that is, two concentric tubes — are shown in Figs. 6 and 7, respectively.

Other properties of the line are shown in Figs. 8, 9, 10 and 11. $Z_1$ and $Z_2$ are the characteristic impedances between conductors 1 and 3 and between 2 and 3, respectively, for the low-loss mode with propagation constant $\gamma_1$. For the high-loss mode, the corresponding impedances are $z_1$ and $z_2$. These are the ratios of voltage and current amplitude for each of the natural traveling waves which can appear on the line. This may be seen clearly when the total line currents and voltages are expressed in terms of the natural traveling waves which are the solutions of the line equations (1)

$$
\begin{aligned}
i_1(x) &= i_{11}e^{\gamma_1 x} + i_{21}e^{-\gamma_1 x} + i_{31}e^{\gamma_3 x} + i_{41}e^{-\gamma_3 x} \\
i_2(x) &= i_{12}e^{\gamma_1 x} + i_{22}e^{-\gamma_1 x} + i_{32}e^{\gamma_3 x} + i_{42}e^{-\gamma_3 x} \\
v_1(x) &= v_{11}e^{\gamma_1 x} + v_{21}e^{-\gamma_1 x} + v_{31}e^{\gamma_3 x} + v_{41}e^{-\gamma_3 x} \\
v_2(x) &= v_{12}e^{\gamma_1 x} + v_{22}e^{-\gamma_1 x} + v_{32}e^{\gamma_3 x} + v_{42}e^{-\gamma_3 x}.
\end{aligned} \tag{9}
$$

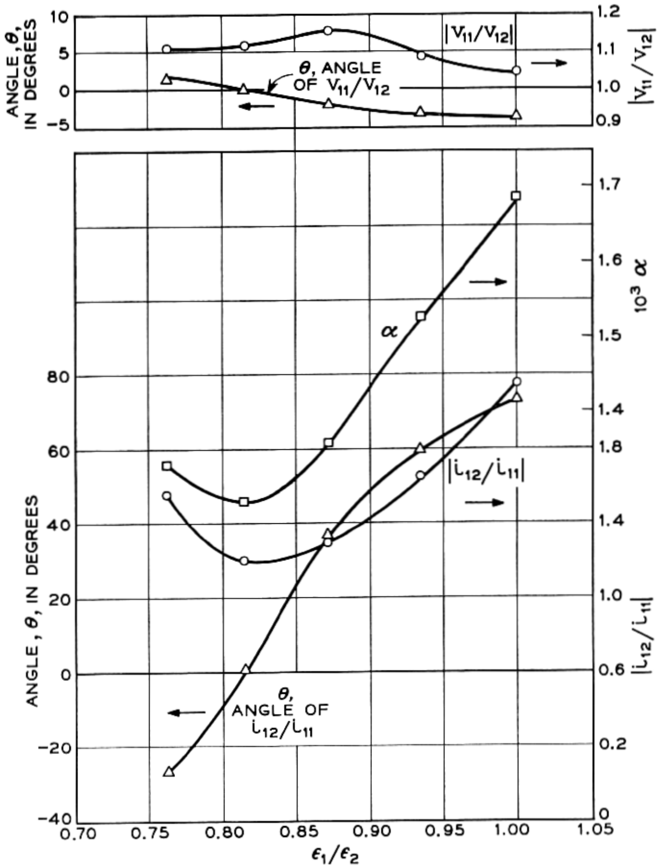


Fig. 5 — Calculated current and voltage ratios in three-conductor Clogston line.

The first subscript refers to the mode and the second to the conductor. For example, $i_{11}$ is the current associated with wave of mode 1 in conductor 1, and $i_{21}$ is that for the wave of mode 2 in conductor 1. Mode 2, as noted in (7), is the backward traveling wave with propagation constant $-\gamma_1$. Thus, modes 1 and 2 are the forward and backward waves having low attenuation $\gamma_1$ and modes 3 and 4 are the forward and backward waves having high attenuation $\gamma_3$. From these facts, we have the following relations

$$
\begin{aligned}
Z_1 &= \frac{v_{11}}{i_{11}} = -\frac{v_{21}}{i_{21}} \\
Z_2 &= \frac{v_{12}}{i_{12}} = -\frac{v_{22}}{i_{22}} \\
z_1 &= \frac{v_{31}}{i_{31}} = -\frac{v_{41}}{i_{41}} \\
z_2 &= \frac{v_{32}}{i_{32}} = -\frac{v_{42}}{i_{42}} \\
h_1 &= \frac{v_{12}}{v_{11}} = \frac{v_{22}}{v_{21}} \\
h_3 &= \frac{v_{32}}{v_{31}} = \frac{v_{42}}{v_{41}}.
\end{aligned} \tag{10}
$$

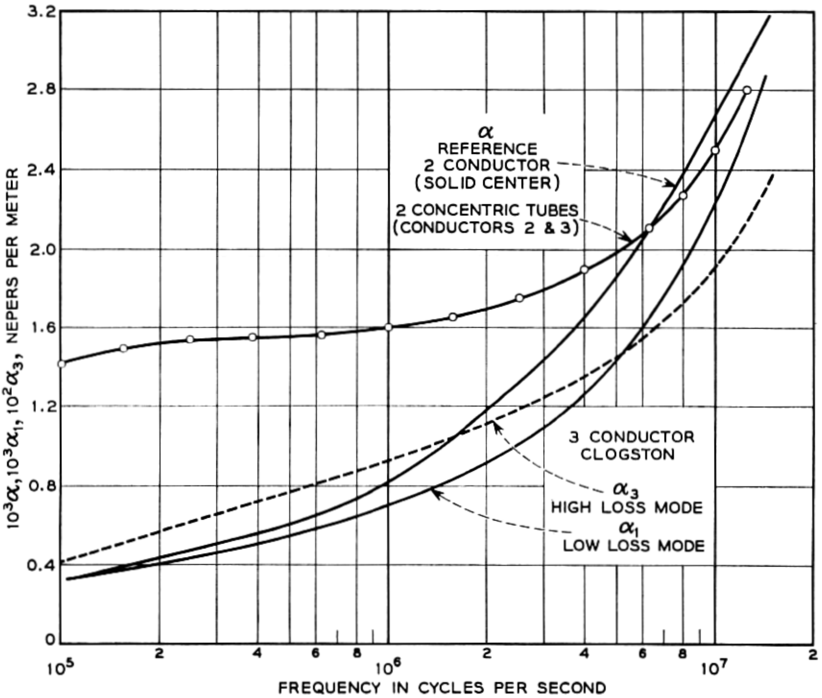


Fig. 6 — Attenuation constants of three-conductor Clogston line compared to those of two two-conductor lines of the same dimensions.

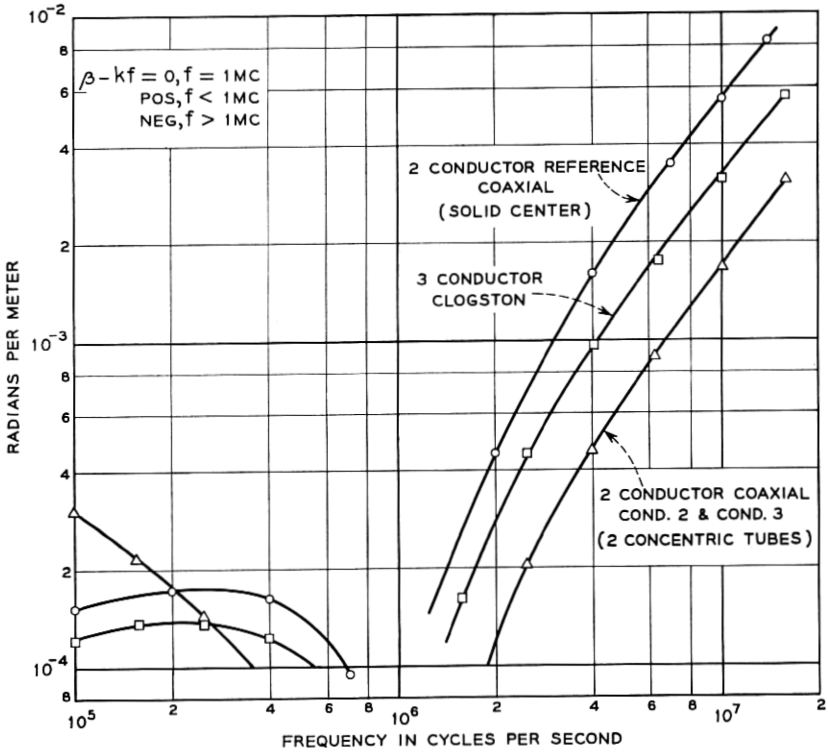


Fig. 7 — Phase distortion of three-conductor Clogston line compared with those of two-conductor coaxials of same dimensions.

The natural ratios of voltages on the two conductors for any one traveling wave are not the same as the ratios of impedances, e.g., $v_{11}/v_{12} \neq Z_1/Z_2$. The voltage and current ratios were calculated by substituting the values of $\gamma_1$ and $\gamma_3$ into (1) and (2). They are plotted in Figs. 9 and 11. From these figures it is seen that the current associated with the high-loss mode (propagation constant $\gamma_3$) in conductor 2 is nearly equal to that in conductor 1 but flows in the *opposite* direction, so that very little flows in the outer conductor 3. Thus, the current that flows in the traveling waves of this mode does not divide more or less evenly between the two inner conductors as it flows down the line, as required for reduction of eddy current losses, but uses these two conductors mainly as go and return paths. In other words, looking at this Clogston line as a two-conductor structure, current in this mode is essentially a circulating current in one of the conductors. It is also seen that below about 2 mc the power at this mode has a fairly large reactive component. The high-loss mode is thus undesirable for other reasons than its order-of-magnitude higher loss.

The relations and figures of this section will be referred to again in Section IV when the subjects of termination and measurement are taken up.

## III. MAKING THE THREE-CONDUCTOR CABLE

After considering a number of ideas and making several trials, it was decided to form the inner dielectric by spiraling a polyethylene thread around the inner solid wire as shown in Fig. 2. This method, suggested by Gordon Raisbeck, seemed to be the most suitable for our experiment. If the thread has a circular cross section, calculations showed that the desired effective dielectric constant is achieved if the threads nearly touch.

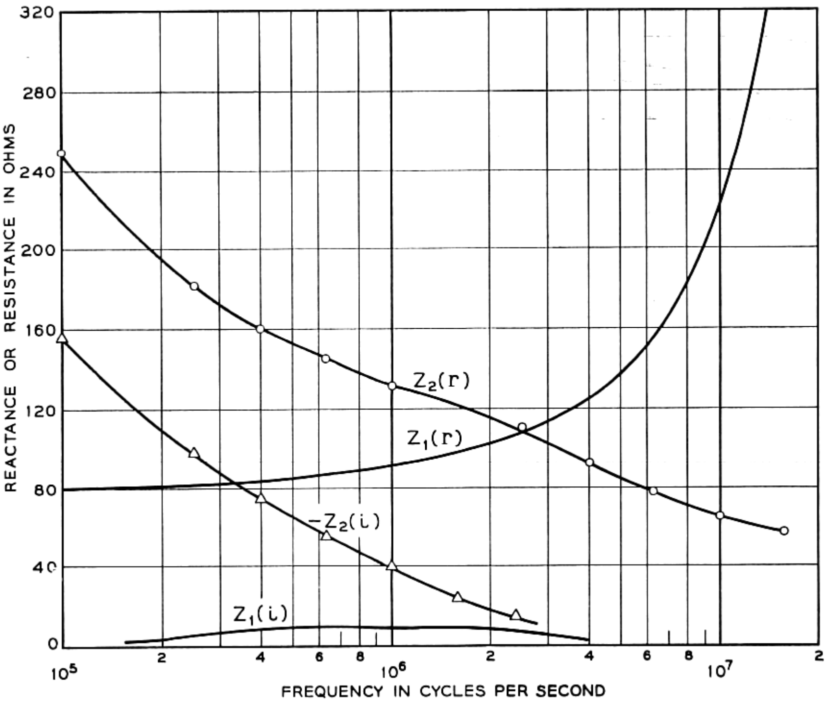


Fig. 8 — Components of characteristic impedances of conductors 1 and 2 (with conductor 3 as return) of three-conductor Clogston line for low-loss mode.

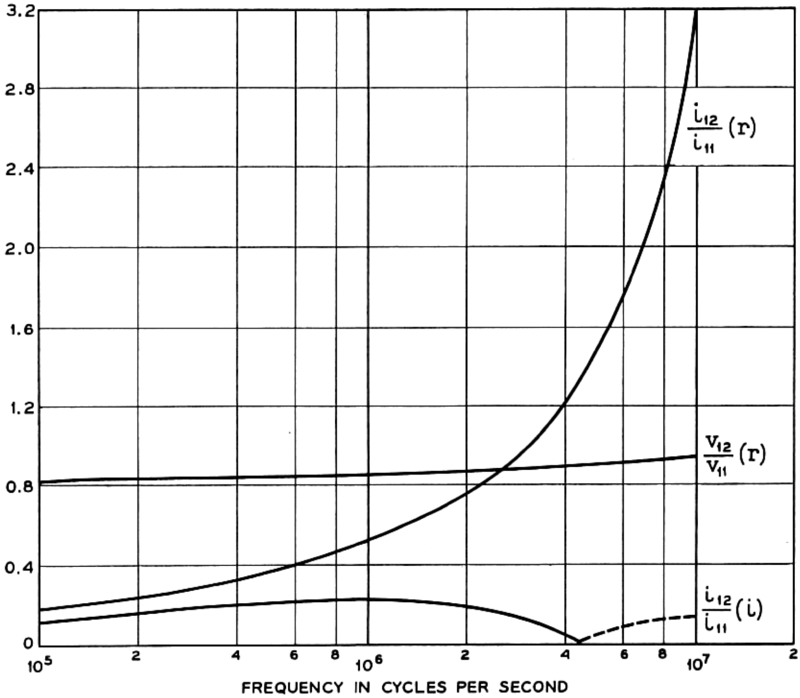


Fig. 9 — Current and voltage ratios in three-conductor Clogston line for low-loss mode.

One problem here was the effect on this insulation of the heat necessarily present during the extrusion of the main dielectric over the composite consisting of conductors 1 and 2 and the dielectric between them. This problem was solved by a modification of the normal extrusion process worked out by members of the chemical research laboratory who did this work for us on their experimental extruder.

The two-conductor composite central member of the cable was made in two trips through a small-scale cable making machine. In the first, 5-mil polyethylene thread was wound around a 19-gauge copper wire as it moved through the machine. In the second trip, 1-mil copper tape and the insulated wire were pulled through a forming die at the center of the spinner so that the copper was wrapped around the insulated wire, leaving a longitudinal gap about 4 mils wide. The copper was held in place by a spiral wrapping of 2-mil thick polyethylene tape. The correct spacing of thread was determined by measuring capacitance and calculating dielectric constant for several pieces a few feet long. A linear puller was used instead of a circular capstan to avoid bending this composite conductor. The puller consisted of two rubber-toothed belts and a suitable mounting and speed control.

The copper tape used for conductor 2 was inspected and cleaned between Teflon blocks. Even so, a few tiny bits of copper came off and lodged between turns of thread. This was detected by an alarm system rigged to give an indication as soon as a low resistance developed between conductors 1 and 2 during the forming process, so that the machine could be stopped and the trouble sought and removed.

An enlarged view (about 8×) of the composite two-conductor center is shown in Fig. 2. The finished form of the structure is at the right, where the polyethylene thread may be seen through the gap between the edges of the copper tape. To the left of this, where the unwrapped polyethylene tape no longer binds it, the gap in the copper tape is

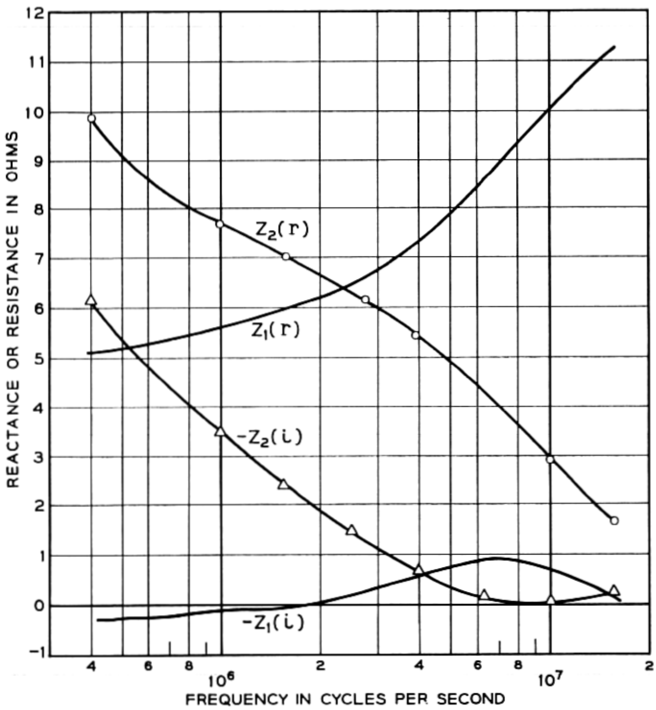


Fig. 10 — Components of characteristic impedances of conductors 1 and 2 (with conductor 3 as return) of three-conductor Clogston line for high-loss mode.

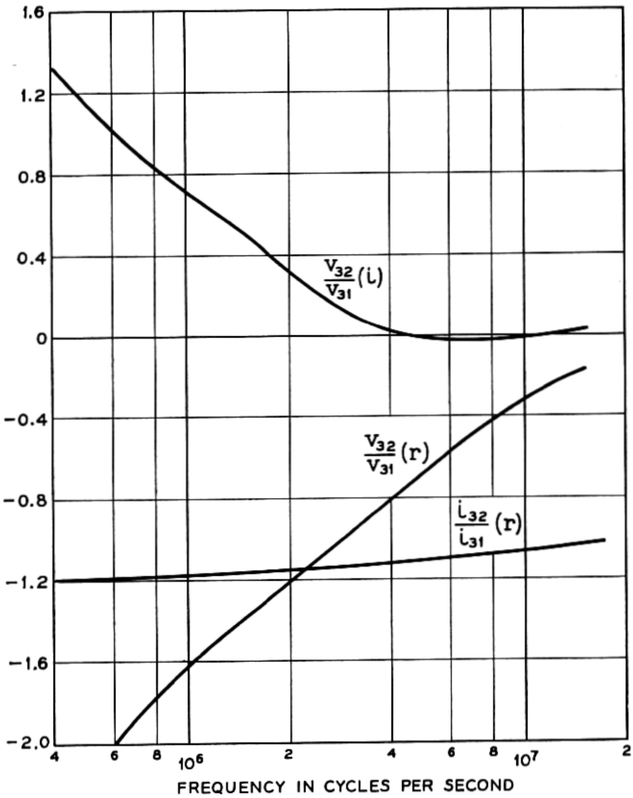


Fig. 11 — Current and voltage ratios in three-conductor Clogston line for high-loss mode.

larger. Small pieces of pressure-sensitive fabric tape have been used to keep the polyethylene thread and tape from unraveling.

Calculations of the dielectric constant of the space between conductors 1 and 2 of this composite were made from capacitance and dimensional measurements; they indicated a value of 0.76 for $\epsilon_1/\epsilon_2$ instead of the intended 0.81. The principal reason for the difference appears to be insufficient control of thread spacing and copper tape binding tightness throughout the whole length of the conductors. Weighed against the difficulties of, and the time required for, making a new central element, the difference between attenuation for 0.76 and that for 0.81 seemed small enough to warrant going ahead with the experiment.

After some preliminary runs, the full length of the central member (about 600 feet) was put through the chemistry laboratory's extruder to make the insulation between conductors 2 and 3 of the cable. This was not entirely successful, since examination showed that the outside diameter of the polyethylene, which was supposed to be 170 mils, actually varied from 150 to 190 mils. After due consideration it was concluded that this was an unavoidable consequence of the modified extrusion process. In order to avoid the uncertain effects of this diameter variation, a machine was made which shaved off excessive polyethylene as the line was pulled along so that the outside diameter now is 151 mils within less than 1 mil. Examination of the inner part of the line at this point showed that no polyethylene had been forced into the space between conductors 1 and 2 during the extrusion process.

The outer conductor of the cable was made by forming 4-mil copper tape over the extruded polyethylene. This conductor is held closed with a small overlap along the longitudinal seam by a spiral wrapping of pressure-sensitive fabric tape.

The length of the cable was carefully measured to be 539.8 feet, or 164.3 meters. The dimensions of the composite structure — conductors 1 and 2 — are difficult to obtain accurately. The best estimates of these dimensions of the line as built, using the terminology of Fig. 3, are (in inches)

$$2a_2 = 0.0466 \qquad 2a_3 = 0.1515$$

$$2b_1 = 0.036$$

$$2b_2 = 0.049 \qquad 2b_3 = 0.1595.$$

Using these dimensions and the measured values of capacitance of the two dielectrics, we find that

$$\epsilon_{r2} = 2.2, \qquad \epsilon_{r1} = 1.67, \qquad \epsilon_1/\epsilon_2 = 0.76.$$

## IV. TERMINATION

Because the answers to the question "How do we terminate the Clogston line?" are complicated, let us try to see first just what is meant by proper termination.

If a two-conductor line with its two natural traveling waves, or modes, is terminated with the characteristic impedance for the forward mode (i.e. $Z_1 = v_{11}/i_{11}$), only the forward traveling wave exists on the line and the attenuation constant of the forward natural wave is the attenuation constant of the transmission line. If the termination is other than $Z_1$, some of the backward natural wave is also present, a situation which may be described otherwise as involving reflection or nonuniform flow of energy.

In the three-conductor line, there are four possible natural traveling waves, and the total currents and voltages on the line are made up of various amounts of these, as indicated by (9). Consideration of these equations along with (10) shows that if the inner conductors 1 and 2 are terminated with the characteristic impedances $Z_1$ and $Z_2$ respectively of the low-loss forward traveling waves and the voltages on these two conductors established in the ratio natural for these waves, then the total currents and voltages in the line will consist of these waves alone. The attenuation of the line will then be that of the low-loss modes, as shown in Figs. 4 and 6. This is the situation we desire, but which was found difficult to achieve experimentally.

In the case of the transposed line, the voltages between the two inner conductors and the outer are the same for the low-loss mode and opposite in sign for the high-loss mode. Thus, the proper way to launch and terminate the line for propagation at the low-loss mode while eliminating the high-loss mode is also the simplest procedure: i.e., to connect together the two inner conductors at both ends of the line.

The situation is not as simple for the elementary Clogston line. The results of calculations plotted on Figs. 8, 9, 10 and 11 show that the voltages of the two inner conductors with respect to the outer are not the same for the low-loss traveling wave. Further, they show that the ratio of these voltages is not the same as the ratio of the characteristic impedances of the two inner conductors when operating in the low-loss mode.

Terminating the Clogston line by connecting together conductors 1 and 2 at the ends and using an average characteristic impedance introduces an unknown proportion of current at the high-loss mode; but the procedure is simple, so it was tried. The attenuation at 1 mc was about 50 per cent higher than expected and about equal to the expected value around 10 mc. Further, there were bumps in the attenuation curve at the low-frequency end, indicating the presence of reflections or some other combination of natural modes. This method is taken up again in the next section, where the results of a search for terminating methods for practical use of the line are given.

The present purpose, however, is to see if the line performs as predicted by theory, and since the most interesting aspect of the theory is the low-loss mode, we tried to establish transmission via this mode alone.

Finally, the method used to drive and terminate the Clogston line shown in Fig. 12 was arrived at. It will be noticed that at the receiving end of the line the two inner conductors are terminated separately. The two-terminal ends of the networks could not be paralleled at both sending and receiving ends of the line without the use of some sort of hybrid

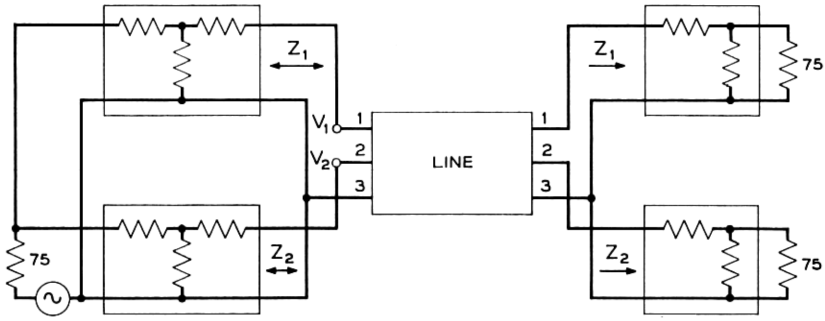


Fig. 12 — Circuit arrangement for terminating and measuring attenuation of three-conductor Clogston line.

or bridge structure; otherwise a transmission loop is formed which makes it very difficult if not out of the question to satisfy all the terminating requirements.

This arrangement, while measuring the attenuation through conductors 1 and 2 separately (which of course should be the same), was adequate for the purpose of testing line performance.

The $T$ and $L$ networks of Fig. 12 were designed to match the impedances $Z_1$ and $Z_2$ of the line and to supply the line input so that the ratio of voltages on conductors 1 and 2 was $v_{11}/v_{12}$. No attempt was made to obtain a design which would provide a match over the frequency band of interest following the data of Figs. 8 and 9. Instead, the networks were made resistive and a separate set was made for each of about eight frequencies. Also, values for $Z_1$, $Z_2$ and $v_{11}/v_{12}$ modified from those shown in Figs. 8 and 9 were used. The latter values were for the expected outer diameter (170 mils) of the cable. When the uneven outer polyethylene had to be shaved down to 151 mils in order to obtain a uniform OD, a complete new calculation was not made; only the change in a few parameters was found, and this used as a basis for estimating the others. The impedances were verified by using a pulse signal and observing for reflections as terminations were varied.

Using launching and terminating networks designed in this way and connected to the line as shown in Fig. 12, the attenuation in both inner conductors was measured at several frequencies surrounding the design frequency. The quantity actually measured was the insertion loss of the line, using a precise transmission measuring set. Some of the data are plotted in Fig. 13, where it may be seen that the two curves of each set intersect near the network design frequency. They cross and diverge in

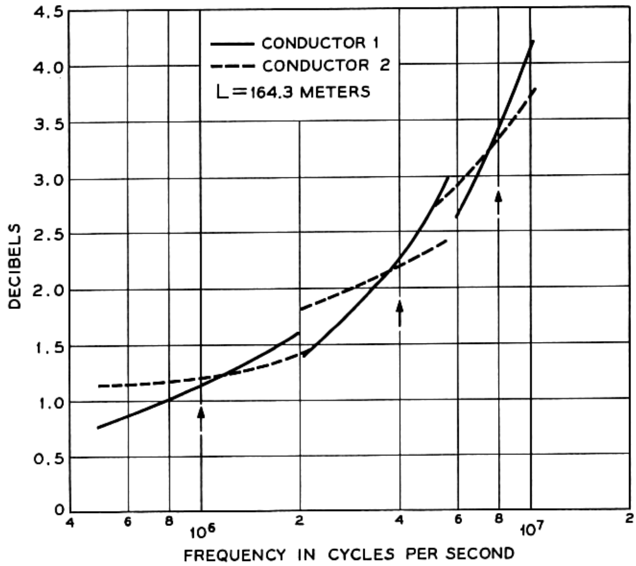


Fig. 13 — Attenuation measurements of three-conductor Clogston line with arrangement of Fig. 12.

both directions beyond the design frequency because the networks provide launching properties only at this frequency. However, they do give the proper value of attenuation in the line at this point. The attenuation at each of the intersections is plotted against frequency in Fig. 14 along with calculated attenuations for the elementary Clogston line and for the reference two-conductor line. The calculated values were obtained by adjusting those shown in Fig. 6 as described previously.

The curves of Fig. 14 show general agreement between measured and predicted attenuation of the Clogston line. Near 4 mc the reduction of attenuation is about 18 per cent instead of the predicted 24 per cent. Part of the reason for this is that, as pointed out in Section III, the ratio of dielectric constants is 0.76 and not 0.815, as required for a minimum. This reduces the predicted change to 21 per cent.

It seems reasonable to conclude that an elementary Clogston line consisting of three coaxial conductors can be built to have a ratio of dielectric constants reasonably near the minimum point and to have an attenuation close to that predicted.

## V. FURTHER STUDY OF THE TERMINATING PROBLEM

The method of termination just described would, of course, be completely unsuitable for practical operation of the line. For this reason, a theoretical study of the problem of coupling the three-conductor line to two-terminal source and load in a practical operation of the line was made by Gordon Raisbeck. This unpublished work led him to a reconsideration of transmission properties of the line when the two inner conductors are connected together at the ends of the line.

Briefly, his conclusions are that for a line long enough so that its total loss is several nepers, the attenuation is very close to that calculated for the low-loss mode of the properly terminated line. Also, the characteristic impedance of the line approaches independence of length as length increases and varies only a small amount with frequency. These facts show that for a reasonable length of line the predicted lower attenuation may be achieved with a simple resistance termination.

Using his formulas, the insertion gain of the 164-meter line when connected between two 50-ohm resistances was computed. This is in fairly good agreement with the corresponding measured loss shown in Fig. 15 when the difference in outside diameters is taken into account. Evidently this is too small a length for the method to be suitable.

Computation from the same formulas of the insertion loss of a 1600-meter length of line (total loss about 2 nepers at 4 mc) was made also and is plotted in nepers per meter in Fig. 16 along with attenuation of

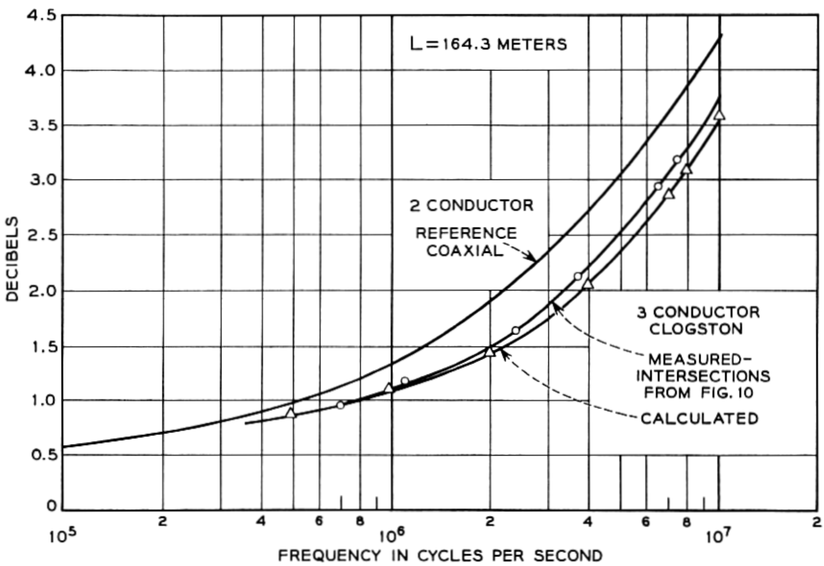


Fig. 14 — Attenuation of three-conductor Clogston transmission line compared with a two-conductor line of like dimensions.

the low-loss mode alone from Fig. 6. This shows that a line of this length with simple termination does have attenuation very close to that of the properly terminated line operating in the low-loss mode. The effects of imperfect launching and impedance matching for the desired mode and the presence of some current in the high-loss mode are thus seen to be small and to occur mainly at the low frequencies. While there is no 1600-meter length of real line available for measurement, the calculated results appear to be reasonable.

A brief resumé of Raisbeck's analysis follows. The method used by him in this work was to start with a length $l$ of the six-terminal, three-conductor line whose transmission properties were calculated and described in Section II and then to short together conductors 1 and 2 at both ends. The matrix describing this new four-terminal structure was then worked out and its transmission properties calculated in terms of two possible traveling waves and characteristic impedances or, in mathematical language, the eigenvalues and eigenvectors of the matrix. The derivation of the matrix of the new line and the calculation of its properties are fairly involved and will not be repeated here.

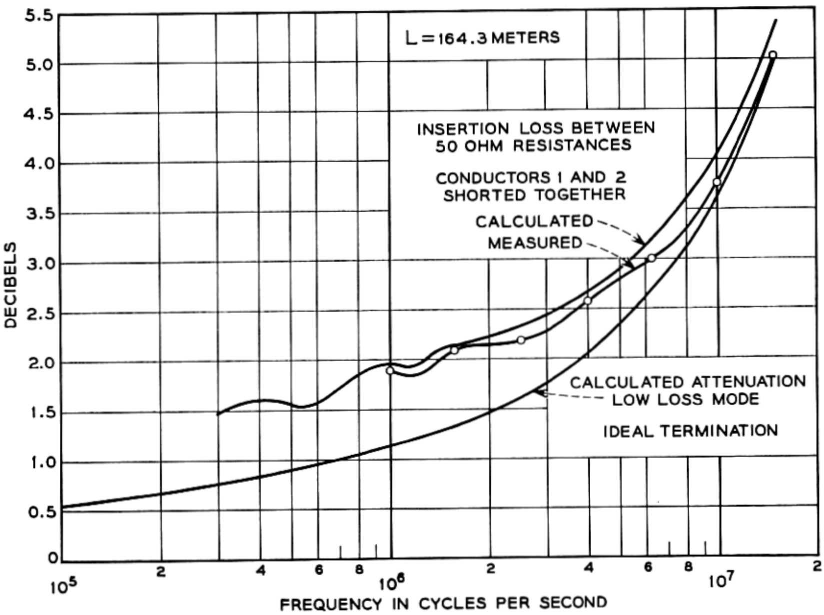


Fig. 15 — Attenuation of short experimental three-conductor Clogston line with simplified termination.

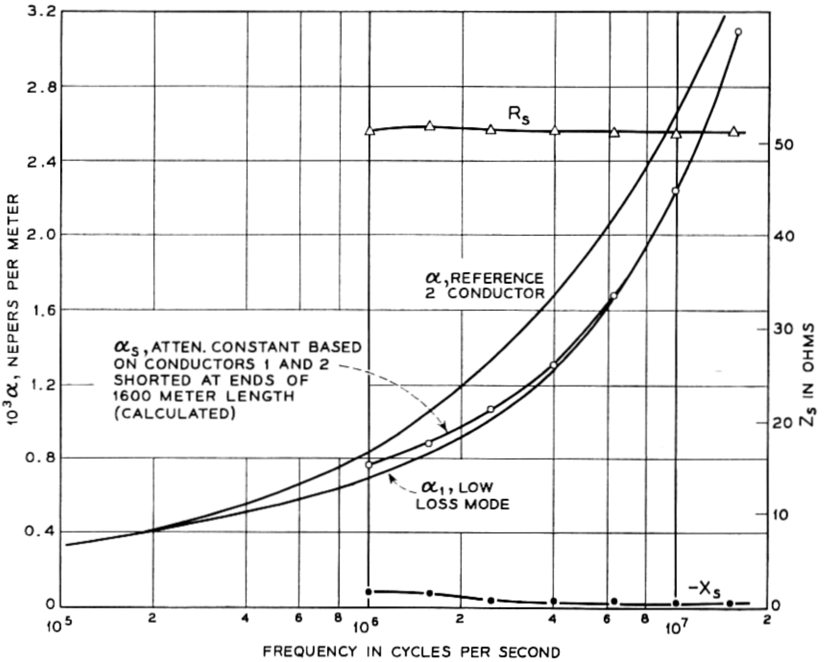


Fig. 16 — Attenuation and characteristic impedance of relatively long three-conductor Clogston line with simplified termination.

The two traveling waves are described by the roots $\lambda_s$ and $\lambda_s^{-1}$ of the characteristic equation of the new matrix. These roots are given by

$$\lambda_s = \frac{1}{2}\frac{a}{b} \pm \frac{1}{2}\sqrt{\frac{a^2}{b^2} - 4} \tag{11}$$

and the characteristic impedance is

$$Z_s = \frac{\pm c/a}{\sqrt{1 - 4b^2/a^2}}, \qquad R_e(Z_s) > 0 \tag{12}$$

where the definitions

$$a = (\lambda_3 - \lambda_3^{-1})(\lambda_1 + \lambda_1^{-1})(-1 + h_3)\left(\frac{1}{Z_1} + \frac{h_1}{Z_2}\right)$$

$$- (\lambda_1 - \lambda_1^{-1})(\lambda_3 + \lambda_3^{-1})(-1 + h_1)\left(\frac{1}{z_1} + \frac{h_3}{z_2}\right)$$

$$b = (\lambda_3 - \lambda_3^{-1})(-1 + h_3)\left(\frac{1}{Z_1} + \frac{h_1}{Z_2}\right) - (\lambda_1 - \lambda_1^{-1})(-1 + h_1)\left(\frac{1}{z_1} + \frac{h_3}{z_2}\right)$$

$$c = (\lambda_1 - \lambda_1^{-1})(\lambda_3 - \lambda_3^{-1})(h_1 - h_3)$$

relate the parameters of the four-terminal line to those of the six-terminal line given in (10). In these expressions

$$\lambda_3 = e^{\gamma_3 l}, \qquad \lambda_1 = e^{\gamma_1 l}, \qquad \lambda_s = e^{\gamma_s l} \tag{14}$$

all the propagation constants $\gamma$ having negative real parts.

After studying these relations, Raisbeck observed that for a line long enough so that the attenuation was several nepers, $\lambda_s$ is nearly equal to $\lambda_1$ ; i.e., the attenuation of the line with conductors 1 and 2 shorted at the ends is approximately the same as that of the low-loss mode in the properly terminated three-conductor line.

To see how this comes about, let $l$ be large enough so that

$$\begin{aligned} \lambda_3 &<<< 1, & \lambda_3^{-1} &>>> 1 \\ \lambda_1 &<< 1, & \lambda_1^{-1} &>> 1 \end{aligned} \tag{15}$$

and then neglect $\lambda_3$ and $\lambda_1$ and divide through by $\lambda_3^{-1}$. We get

$$\begin{aligned} \frac{a}{b} &= \lambda_1^{-1}\left[1 - \frac{(-1 + h_1)\left(\frac{1}{z_1} + \frac{h_3}{z_2}\right)}{(-1 + h_3)\left(\frac{1}{Z_1} + \frac{h_1}{Z_2}\right)}\right] \\ &= \lambda_1^{-1}(1 - d) \end{aligned} \tag{16}$$

and

$$\frac{c}{a} = \left(\frac{Z_1Z_2}{Z_2 + h_1Z_1}\right)\left(\frac{h_1 - h_3}{1 - h_3} \Big/ (1 - d)\right). \tag{17}$$

The quantity $d$ does not depend on length of line, but on the voltage ratios and impedances of the natural traveling waves of the line. It is small, being 0.1 at 1 mc, 0.056 at 4 mc, and 0.016 at 10 mc, using the data of Figs. 8, 9, 10 and 11. Thus $a/b$ is large, and we have very nearly

$$\lambda_s^{-1} \doteq \frac{a}{b} \doteq \lambda_1^{-1}(1 - d)$$

$$\lambda_s \doteq \frac{b}{a} \doteq \lambda_1(1 + d) \tag{18}$$

or

$$e^{\gamma_s l} \doteq e^{\gamma_1 l}(1 + d).$$

This shows that the attenuation of the line having conductors 1 and 2 shorted at the ends of a sufficiently long section is nearly the same as that of the ideally terminated line. The characteristic impedance of the line with conductors 1 and 2 shorted is given by

$$Z_s \doteq \frac{c}{a} \doteq \left(\frac{Z_1 Z_2}{Z_2 + h_1 Z_1}\right)\left(\frac{h_1 - h_3}{1 - h_3}\right)(1 + d). \tag{19}$$

This expression for $Z_s$ is nearly equal to the impedance of $Z_1$ and $Z_2$ in parallel.

While the values of $\alpha_s$ and $Z_s$ plotted in Fig. 16 were calculated from the exact values of $a$, $b$, and $c$, the calculations show that for the 1600-meter length, the above approximations are quite close to the correct values.

The formula for insertion gain between two resistances $R$, as used in the calculations, is

$$\begin{aligned} G &= \frac{4RZ_s}{\lambda_s^{-1}(Z_s + R)^2 - \lambda_s(Z_s - R)^2} \\ &\doteq \frac{4R/Z_s}{(1 + R/Z_s)^2}\, e^{-\gamma_s l}. \end{aligned} \tag{20}$$

## VI. SUMMARY

Early in the paper, the properties of an elementary Clogston line were discussed, and the process of building such a line for experiment described. In testing the line's performance, it was found that it performs about as predicted and that terminating the line ideally is a difficult problem. In Section V, it has been seen that Raisbeck's analysis leads to a simple way of coupling the line to source and load with resistances which, while it does not provide ideal termination, does allow performance which is quite close to ideal when the length of line section involved has a loss of around 20 db or more.

Thus we conclude that it is possible to build a uniform three-conductor line which has about 20 per cent less attenuation than a comparable two-conductor line over a considerable frequency band. Since this is almost the same as the reduction obtained with the transposed line, this line is an improvement in not requiring transpositions.

While a line made of two concentric tubular conductors may be the best for pulse transmission because of its low phase distortion, the use of this three-conductor Clogston line may be thought of as a good compromise. This is because it provides less phase distortion than a conventional two-conductor line, although more than the tubular line (see Fig. 7), and in addition its solid center conductor permits a low-loss path for sending power down the line, and lowers the transmission loss considerably in the low-frequency range (see Fig. 6).

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