NEW COMPUTER LIBRARY ROUTINE B4-SQR1-26

TITLE: Square Root

TYPE: closed, relocatable, mnemonic

LENGTH: 9 words

TEMPORARY STORAGE: 3 words at fixed memory locations 0,1,2

DURATION: approximately 200 microseconds (Oct 62)

FAST REGISTERS CHANGED: none

ACCURACY: relative error $< 3 \times 2^{-44}$

PARAMETERS: link in M15

USE: This subroutine replaces the number A in the accumulator by

$$\sqrt{A} \quad \text{if } A \geq 0,$$

leaves A unchanged and sets OV if A < 0.

DESCRIPTION: $\sqrt{A}$ is found by applying 4 steps of the algorithm

$$a_{n+1} = (1/2)(A/a_n + a_n) \ .$$

if

$$A = x4^y, \quad 1/4 \leq x < 1,$$

the initial guess a_o is

$$a_o = (1/2)(1+x)\,2^y = \begin{cases} (1/2)(1+x)\,4^{y/2} & \text{if } y \text{ even} \\ (1+x)\,4^{(y-1)/2} & \text{if } y \text{ odd} \end{cases}$$

DATE: October 30, 1962

PROGRAMMED BY: J. Nievergelt

APPROVED BY:

0	JDC3,1,8R	Jump if A < 0, to produce OV
	JDC5,2,8R	Jump if A = 0, to exit
1	SFR7,0	A normalized
	STR8,3,1	
	SEX15,0	X as floating point number in accumulator
2	CAE8,3,0	
	ADD9,3,1	1 + x
3	CRM15,2,1	y/2 or (y − 1)/2 in M15
	JNM15,2,4R	Jump if y was odd
4	MPY10,3,2048	If y was even, (1 + x)/2
	ADE15,0	
	CSM14,2,4	Set counter
5	STR8,3,2	A$_n \to$ memory
	VID8,3,1	A/A$_n$
6	ADD8,3,2	+ A$_n$
	MPY10,3,2048	× 1/2
7	CJU14,1,5R	
	LFR7,0	
8	JLH15,0	Produce OV
	DIV15,3	
	JLH15,0,0	