

UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

AUXILIARY LIBRARY ROUTINE M 32 - 298

**TITLE:** Williams Memory Routine for Linear Programming by  
the Simplex Method-Modified for Variable  $C_j$   
**TYPE:** Complete Floating Point Program  
**DESCRIPTION:** This is the same as Library Routine M 15 - 183 "Williams  
Memory Routine for Linear Programming by the Simplex  
Method", except that one of the  $C_j$ 's may be varied  
by adding an increment  $\Delta C_j$ .

All directions are the same as for M 15 - 183 except:  
Prepare the parameter tape as follows:

005+  
00F 00jF  
00F 00iF  
0019+  
00F 00tF  
00kF 00bF  
24999N

where

$j$  = number of activities (Excluding slack and  $P_0$ )  
 $i$  = number of restraints  
 $t$  = frequency of intermediate print-out  
 $k$  = designation of  $C_k$  to be varied  
 $b = 0$  for minimization problem 1 for maximization  
problem

In addition, two parameters are needed at the end of the  
data tape. These should be in floating decimal representa-  
tion. The first specifies the increment  $\Delta C_k$  by which  $C_k$   
is to be increased for each separate solution. The second  
parameter at the end of the tape specifies the upper limit  
desired for  $C_k$ .

The program proceeds by computing an optimal solution for the initial value, then it adds  $\Delta C_k$  to  $C_k$  and computes a new optimal solution for  $C_k' = C_k + \Delta C_k$ . (The new solution may or may not be different from the previous one). It continues in this way until it has computed a solution for  $C_k$  at its upper limit.

The print-out will be headed by the  $C_k$  used in computing that particular solution. The parameter  $t$  controls the intermediate print-out, if any, between successive final solutions.

**TIMING:**

The computational procedure is the same as for M 15 - 183, so a comparable amount of time will be required per iteration. However, there will in general be more iterations. The time required to change the  $C_k$  is negligible.

**CAPACITY:**

$$4(i + j + 1) + i j \leq 682$$

**EXAMPLE:**

Minimize:

$$Z = 21y_1 + 24y_2 + c_3 y_3$$

where  $C_3$  varies from  $2 \frac{1}{2}$  to  $10 \frac{1}{2}$  and  $\Delta C_3 = 4$  according to the restraints.

$$\begin{aligned} \text{(a)} \quad y_1 + 2y_2 + y_3 &\geq 2 \\ 3y_1 + 3y_2 + y_3 &\geq 4 \end{aligned} \quad y_i \geq 0$$

then (a) can be rewritten:

$$\begin{aligned} \text{(b)} \quad y_1 + 2y_2 + y_3 - y_4 &= 2 \\ 3y_1 + 3y_2 + y_3 - y_5 &= 4 \end{aligned} \quad y_i \geq 0$$

where  $C_4 = C_5 = 0$ .

In vector form this is

$$\text{(c)} \quad \begin{bmatrix} 1 & 2 & 1 & -1 & 0 \\ 3 & 3 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 2 \\ - \\ 4 \end{bmatrix}$$

but this contains no unit vectors, so (c) is written as:

$$(d) \begin{bmatrix} 1 & 2 & 1 & -1 & 0 & 1 & 0 \\ 3 & 3 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_7 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

and

$$Z = 21y_1 + 24y_2 + c_3y_3 + My_6 + My_7$$

Now if M is set equal to  $9 \times 10^5$ , for example, then it is intuitively obvious that in order for Z to be a minimum,  $y_6 = y_7 = 0$  and thus (c) and (d) would be in a sense, "equivalent".

Then

$$p_0 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, p_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, p_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, p_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$p_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, p_5 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

and  $p_6$  and  $p_7$  are the basis vectors.

The coefficient tape is:

$\left. \begin{array}{l} +2+01 \\ +4+01 \end{array} \right\} p_0$	$+0+00 \quad c_0$
$\left. \begin{array}{l} +1+01 \\ +3+01 \end{array} \right\} p_1$	$\left. \begin{array}{l} +21+02 \\ +24+02 \end{array} \right\} c_1 \text{ and } c_2$
$\left. \begin{array}{l} +2+01 \\ +3+01 \end{array} \right\} p_2$	$+25+01 \quad \text{initial } c_3$
$\left. \begin{array}{l} +1+01 \\ +1+01 \end{array} \right\} p_3$	$\left. \begin{array}{l} +0+00 \\ +0+00 \end{array} \right\} c_4 = c_5 = 0$
$\left. \begin{array}{l} -1+01 \\ +0+00 \end{array} \right\} p_4$	$\left. \begin{array}{l} +9+05 \\ +9+05 \end{array} \right\} c_6 = c_7 = 9 \times 10^5$
$\left. \begin{array}{l} +0+00 \\ -1+01 \end{array} \right\} p_5$	$+4+01 \quad \Delta c_3 = 4$
	$+105+02 \quad \text{terminal } c_3$

The parameter tape would be

005+  
00F 005F (5 structural vectors)  
00F 002F (2 basis vectors)  
0019+  
00F 001F (t = 1)  
003F 000F to indicate  $C_3$  varies  
24999N and Z is to be minimized.

The output has the form:

+250000000 +01  $C_3$   
  
006 +200000000 +01  
006 +400000000 +01  
  
000 +540000000 +06  
001 +359979000 +06  
002 +449976000 +06  
003 +179997500 +06  
004 -900000000 +05  
005 -900000000 +05  
  
002 +100000000 +01  
007 +100000000 +01  
  
000 +900240002 +05  
001 +134991000 +06  
006 -224988000 +06  
003 -449904999 +05  
004 +134988000 +06  
005 -900000000 +05  
  
002 +666666666 +00  
001 +666666668 +00

000	+300000000	+02
007	-899940000	+05
006	-899970000	+05
003	+650000000	+01
004	<del>-29</del> 9999999	+01
005	-600000000	+01
003	+100000000	+01
001	+100000000	+01
000	+235000000	+02
007	-899907500	+05
006	-900067500	+05
002	-975000001	+01
004	+675000000	+01
005	-925000001	+01
003	+400000000	+01
004	+200000000	+01
000	+100000000	+02
007	-899975000	+05
006	-900000000	+05
002	+650000000	+02
001	-135000000	+02
005	-250000000	+01
	+650000000	+01 C <sub>3</sub>
003	+400000000	+01
004	+200000000	+01
000	+260000000	+02
007	-899935000	+05
006	-900000000	+05
002	-449999999	+01
001	-149999999	+01
005	-650000000	+01

+105000000 +02 C<sub>3</sub>

003 +400000000 +01

004 +200000000 +01

000 +420000001 +02

007 -899895000 +05

006 -900000000 +05

002 +750000004 +01

001 +105000001 +02

005 -105000000 +02

003 +100000000 +01

001 +100000000 +01

000 +315000000 +02

007 -899947500 +05

006 -899947500 +05

002 +225000002 +01

004 -525000001 +01

005 -524999999 +01

002 +666666667 +00

001 +666666667 +00

000 +300000000 +02

007 -899940000 +05

006 -899970000 +05

003 -150000001 +01

004 -300000000 +01

005 -600000000 +01

DATE	<u>May 26, 1960</u>
CODED BY	<u>L. Isaacson</u>
WRITE-UP BY	<u>Richard Hacker</u>
APPROVED BY	<u>J. Snyder</u>

LOCATION	ORDER	NOTES	PAGE 1	M 32
	DOI Routine X 1			
	003K			
0	00 F			
	00 7F			
1	00 F			
	00 173F			
	0012K			
0	00 F			
	00 1S5			
	0016K			
0	00 341F			
	00 341F			
1	00 S6			
	00 S6			
2	00 F			
	00 341F			
	0021K			
0	50 12F			
	75 6F			
1	S5 F			
	L4 12F			
2	L4 6F			
	40 7F			
3	L0 12F			
	L0 6F			
4	L4 18F			
	40 9F			
5	L4 12F			
	40 10F			
6	L4 6F			
	40 14F			
7	L4 12F			
	40 15F			
8	L4 6F			
	40 11F			

LOCATION	ORDER	NOTES	PAGE 2	M 32
9	L4 12F 40 13F			
10	L4 12F 40 8F			
11	50 12L 26 999F			
12	00 F 00 L			
13	00 F 26 L 261N			
14	L1 6F L0 5F			
15	40 8L 41 SF			
16	F5 SF 40 1SF			
17	F5 8L 40 8L			
18	36 9L L5 2L			
19	L4 7L 40 2L			
20	26 2L 00 F			
21	00 1F 00 1F			
22	00 F 00 F			
23	50 10L 26 999F			
24	00 F 00 L			
25	00 F 26 L			



LOCATION	ORDER	NOTES	PAGE 3	M 32
	261N			
26	41 11F			
	15 16F			
27	42 4L			
	50 1L			
28	26 S4			
	OK SN			
29	1K S6			
	8K F			
30	8S 3F			
	15 F			
31	17 SK			
	84 3F			
32	8S 3F			
	12 4L			
33	85 3F			
	00 S9			
34	0S SS			
	8J 69L			
35	03 3L			
	2K S5			
36	8K F			
	8S 4F			
37	21 1SS			
	80 4F			
38	83 14L			
	21 1SS			
39	8S 4F			
	8J 71L			
40	8J 73L			
	23 11L			
41	85 4F			
	82 141L			
42	8J 75L			
	3K S6			

LOCATION	ORDER	NOTES
43	8K F	
	3N F	
44	83 25L	
	31 F	
45	83 25L	
	83 25L	
46	31 341F	
	36 F	
47	8S 3F	
	84 120L	
48	82 23L	
	81 3F	
49	83 25L	
	81 3F	
50	8S 120L	
	8J 81L	
51	8J 73L	
	33 17L	
52	85 120L	
	80 121L	
53	82 65L	
	82 65L	
54	4K S6	
	45 F	
55	4S S8	
	42 28L	
56	8J 85L	
	5K S6	
57	8K F	
	5S F	
58	53 31L	
	8K 1F	
59	82 33L	
	8S F	
60	6K SN	
	85 F	

LOCATION	ORDER	NOTES PAGE 5 M 32
61	86 20F	
	6S SJ	
62	8J 93L	
	62 34L	
63	3K SN	
	4K S6	
64	41 S8	
	37 SJ	
65	44 F	
	4S F	
66	43 38L	
	8J 95L	
67	32 37L	
	5K SN	
68	55 SJ	
	8S F	
69	8J 97L	
	53 42L	
70	8J 99L	
	2K S6	
71	8F 1F	
	8J 56L	
72	25 341F	
	89 9F	
73	22 45L	
	22 45L	
74	8F 1F	
	7K SN	
75	8J 60L	
	75 SS	
76	89 9F	
	73 49L	
77	73 49L	
	8J 64L	
78	0F F	
	50 52L	

LOCATION	ORDER	NOTES
79	26 S4	
	4K S7	
80	88 F	
	4S 341F	
81	43 54L	
	8J 122L	
82	92 131F	
	L5 SL	
83	00 28F	
	82 12F	
84	92 963F	
	F5 56L	
85	40 56L	
	26 29S4	
86	92 131F	
	L5 SF	
87	00 28F	
	82 12F	
88	92 963F	
	F5 60L	
89	40 60L	
	26 29S4	
90	L1 19F	
	40 12F	
91	26 83L	
	8J 66L	
92	FF F	
	00 52L	
93	L5 66L	
	42 51L	
94	26 116L	
	8J 67L	
95	L5 4L	
	L4 6F	
96	40 4L	
	26 29S4	

LOCATION	ORDER	NOTES PAGE 7 M 32
97	F5 11F	
	40 13F	
98	26 29S4	
	00 F	
99	F5 11F	
	40 11F	
100	26 29S4	
	00 F	
101	50 13F	
	75 6F	
102	L5 16F	
	S4 F	
103	42 17L	
	42 18L	
104	42 20L	
	42 28L	
105	42 31L	
	41 11F	
106	26 114L	
	00 F	
107	L5 11F	
	40 18F	
108	26 29S4	
	00 F	
109	41 S3	
	50 83L	
110	26 S4	
	82 16L	
111	L5 28L	
	L4 18F	
112	42 33L	
	42 89L	
113	L5 16F	
	L4 18F	
114	42 34L	
	42 42L	

LOCATION	ORDER	NOTES PAGE 8 M 32
115	22 89L L5 F	
116	40 20F L5 16F	
117	42 39L 46 39L	
118	26 29S4 00 F	
119	L5 34L L4 6F	
120	40 34L 26 29S4	
121	L5 39L L4 17F	
122	40 39L 26 29S4	
123	L5 42L L4 6F	
124	40 42L 26 29S4	
125	L5 9F L4 13F	
126	42 103L 42 105L	
127	L5 10F L4 18F	
128	42 104L 42 106L	
129	22 103L 50 F	
130	22 104L L5 F	
131	22 105L 40 F	
132	S5 F 40 F	

LOCATION	ORDER	NOTES PAGE 9 M 32
133	L1 53L 40 53L	
134	36 L L5 14F	
135	L4 13F 42 103L	
136	42 105L L5 15F	
137	L4 18F 42 104L	
138	42 106L L5 121L	
139	40 120L 22 103L	
140	26 29S4 00 F	
141	36 116L 26 29S4	
142	L5 14F 42 60L	
143	L5 15F 42 56L	
144	41 S3 50 118L	
145	26 S4 82 44L	
146	40 F 00 127F	
147	40 F 00 127F	
148	92 135F 50 122L	
149	26 S4 88 F	
150	8S 149L 88 F	

LOCATION	ORDER	NOTES PAGE 10 M 32
151	8S 150L 85 F	
152	89 9F 8J L	
153	92 143F 92 515F	
154	L5 14F 42 130L	
155	41 F L5 93F	
156	42 F L5 F	
157	L0 F 40 1F	
158	L3 1F 32 134L	
159	F5 130L 42 130L	
160	22 130L L5 130L	
161	L0 14F L4 9F	
162	42 138L 42 139L	
163	22 137L 50 137L	
164	26 S4 85 F	
165	84 149L 8S F	
166	8S 151L 89 9F	
167	8J L 85 151L	
168	80 150L 82 68L	



LOCATION	ORDER	NOTES	PAGE 11	M 32
169	8J 144L 00 F			
170	L5 145L 42 51L			
171	26 116L 00 146L			
172	L5 148L 42 51L			
173	L1 19F 40 12F			
174	26 127L 00 64L			
	00173K			
0	L1 19F 32 2L			
1	40 12F L5 11L			
2	40 135F L5 20F			
3	10 20F 42 93F			
4	L4 9F 42 146F			
5	47 20F L1 20F			
6	36 9L L5 12L			
7	40 32F L5 13L			
8	40 33F 26 9L			
9	50 10L 26 999F			
10	00 F 00 L			

LOCATION	ORDER	NOTES	PAGE 12	M 32
11	F5 12F 40 12F			
12	25 1SS 80 4F			
13	83 35F 25 1SS			
14	00 F 26 L 261N			
15	00 59F L0 166L			
16	L0 124L 40 2L			
17	50 F S5 20F			
18	40 2F 32 150L			
19	46 6L 10 12F			
20	L4 125L 46 9L			
21	L5 F 10 1F			
22	40 F 10 6F			
23	01 7F 40 1F			
24	26 F 40 F			
25	L5 1S3 L0 1F			
26	50 F 32 15L			
27	40 2F S5 F			

LOCATION	ORDER	NOTES PAGE 13 M 32
28	50 S3	
	40 S3	
29	L5 1F	
	40 1S3	
30	L1 2F	
	L4 125L	
31	42 17L	
	L0 126L	
32	36 29L	
	7J F	
33	L4 S3	
	40 S3	
34	LL S3	
	36 27L	
35	50 S3	
	7J 32L	
36	40 S3	
	L5 43L	
37	L4 1S3	
	40 1S3	
38	26 27L	
	50 S3	
39	75 127L	
	00 4F	
40	40 S3	
	L1 43L	
41	L4 1S3	
	40 1S3	
42	L3 S3	
	36 68L	
43	L4 42L	
	32 23L	
44	L5 2L	
	36 1L	
45	L4 128L	
	22 1L	

LOCATION	ORDER	NOTES	PAGE 14	M 32
46	7L 4095F LL 4095F			
47	00 F 00 1000 0000 0000 J			
48	00 F 00 1000 0000 000 J			
49	00 F 00 1000 0000 00 J			
50	00 F 00 1000 0000 0 J			
51	00 F 00 1000 0000 J			
52	00 F 00 1000 000 J			
53	00 F 00 1000 00 J			
54	00 F 00 1000 0 J			
55	00 F 00 1000 J			
56	00 F 00 64F			
57	00 F 00 4999 9999 998 J			
58	00 F 00 1F			
59	L1 F 22 9L			
60	L1 F 26 67L			
61	L5 131L 26 133L			
62	L5 131L 22 133L			
63	J0 1021F 26 10L			

LOCATION	ORDER	NOTES PAGE 15 M 32
64	L5 F 26 67L	
65	L1 1F 26 63L	
66	L0 41L 26 60L	
67	49 1F 22 77L	
68	L5 6L 22 97L	
69	41 1F 26 138L	
70	L5 6L 26 69L	
71	L7 S3 26 76L	
72	L5 2F 22 5L	
73	92 131F 26 95L	
74	L5 6L 22 149L	
75	L4 1S3 40 1S3	
76	50 F 7J S3	
77	00 1F 22 18L	
78	L4 130L L4 1S3	
79	40 1S3 50 42L	
80	75 S3 66 F	
81	S5 31L 22 18L	

LOCATION	ORDER	NOTES	PAGE 16	M 32
82	40 S3 L5 1F			
83	40 1S3 26 29L			
84	46 75L L5 1S3			
85	32 71L 41 S3			
86	41 1S3 10 167L			
87	34 72L L5 S3			
88	10 6F 50 1F			
89	00 7F 14 1S3			
90	40 F 26 29L			
91	40 S3 L3 F			
92	22 9L 81 4F			
93	00 39F 36 80L			
94	11 1F 40 1F			
95	27 84L 50 1F			
96	7J 32L 40 1F			
97	50 1F 75 2F			
98	00 39F 14 S3			
99	40 S3 81 4F			

LOCATION	ORDER	NOTES
100	40 2F	
	L0 129L	
101	36 87L	
	22 80L	
102	81 4F	
	40 F	
103	81 4F	
	50 F	
104	74 129L	
	L1 2F	
105	L4 129L	
	36 92L	
106	S1 31L	
	22 92L	
107	S5 F	
	L4 41L	
108	40 1S3	
	26 19L	
109	L1 F	
	50 4F	
110	L5 2F	
	10 20F	
111	42 94L	
	47 94L	
112	26 29L	
	L4 91L	
113	46 105L	
	L5 94L	
114	L0 79L	
	36 101L	
115	92 963F	
	22 122L	
116	46 94L	
	L5 S3	

LOCATION	ORDER	NOTES
117	40 F 36 104L	
118	92 708F 22 104L	
119	92 644F 50 101L	
120	7J F L6 F	
121	L6 F 32 110L	
122	L5 43L L4 1S3	
123	40 1S3 L5 42L	
124	40 S3 40 F	
125	22 104L 10 39F	
126	75 129L 00 36F	
127	82 4F 10 40F	
128	L5 6L L0 49L	
129	46 6L L0 62L	
130	36 111L 92 961F	
131	L1 48L 40 48L	
132	32 118L 92 961F	
133	26 29L 46 6L	



LOCATION	ORDER	NOTES
134	L5 1S3 L0 41L	
135	40 F 50 F	
136	75 33L 00 38F	
137	26 102L 92 513F	
138	00 24F 26 101L	
139	5S F S5 F	
140	00 172L 00 31L	
141	00 172L 00 42L	
142	50 F 00 F	
143	5S 1F S5 F	
144	00 F 00 10F	
145	00 F 00 65F	
146	50 F S5 20F	
147	00 1F 00 1F	
148	L0 124L 40 1F	
149	L5 2F 46 1F	
150	32 143L L5 S3	

LOCATION	ORDER	NOTES
151	36 137L 26 29L	
152	L5 1F 22 1L	
153	L5 2F 36 146L	
154	46 1F 50 1F	
155	75 33L 00 18F	
156	40 S3 L5 130L	
157	L4 43L 40 1S3	
158	26 19L L5 157L	
159	L4 132L 40 157L	
160	00 28F 26 136L	
161	10 20F 42 1F	
162	L1 1F 50 158L	
163	L4 157L 40 157L	
164	26 29L 46 157L	
165	26 29L 50 2F	
166	01 3F L4 147L	
167	42 143L 42 155L	
168	42 148L 42 149L	

LOCATION	ORDER	NOTES PAGE 21 M 32
169	42 144L	
	JO 166L	
170	11 3F	
	L5 157L	
171	S4 F	
	26 4L	
172	80 F	
	00 2048F	
173	80 F	
	00 F	
174	80 F	
	00 F	
175	80 F	
	00 F	
176	80 F	
	00 F	
177	80 F	
	00 F	
178	80 F	
	00 F	
179	80 F	
	00 F	
180	80 F	
	00 F	
181	LL 4094F	
	4K 4076F	
182	00 F	
	00 128F	
	2073N	