UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE S 3 - 130

TITLE	Logarithm (DOI or SADOI)
TYPE	Closed subroutine with standard entry
NUMBER OF WORDS	14
TEMPORARY STORAGE	0, 1, 2
ACCURACY	Maximum error 2^{-38} assuming exact argument. Has biased error from + 0 to -2^{-39}.
DURATION	Approximately 51 milliseconds
DESCRIPTION	This routine is entered with the argument x in A, x being within the limits $0 < x < 1$. $\text{Log}_2\, x$ is left in AQ with the integer part in A and the fractional part in Q with $q_0 = I$. If x = 0 the routine will loop for determining the integer part of the logarithm and thus if it is possible that x = 0 this should be tested for before entering the routine.

This result may be converted to any desired base with the use of the formula $\log_a x = \log_2 x \log_a 2$. The two most often used conversion factors are:

$$\log_e 2 = 0.69314\ 71805\ 60$$
$$\log_{10} 2 = 0.30102\ 99956\ 64$$

Since $\log_2 x$ is left in AQ, a right shift may be necessary to bring the integer part of the logarithm into Q before multiplying by the conversion factor. In this process q_0 remains equal to one and thus the logarithm has the correct sign. In general, for $0 \leq x < 1$, it will be sufficient to shift right 6 times since the integer part can be no less than -39.

The direct conversion factors after shifting right six places for base e and base 10 respectively are:

$$(64/100) \log_e 2 = 0.44361\ 41955\ 58$$
$$(64/100) \log_{10} 2 = 0.19265\ 91972\ 25$$

thus leaving $(1/100) \log_e x$ or $(1/100) \log_{10} x$ in the accumulator after multiplying.

The two most significant decimal digits are interpreted as the integral part of the logarithm.

If $1/2 \leq x < 1$, the correct result is in Q alone when leaving the routine and no shifting is necessary before base conversion. More accuracy can thus be retained in the final result.

MATHEMATICAL METHOD

The quantity $\log_2 x$ is found in this routine to take advantage of the binary operation of the computer. The integer part of $\log_2 x$ is found by considering x as $2^{-m}(w)$ where $0 < x < 1$, $1/2 \leq w < 1$, and m is a positive integer. Then

$$\log_2 x = \log_2 2^{-m}(w) = -m + \log_2 w$$

$\log_2 w$ is fractional and $-m$ is the integer part of $\log_2 x$. The quantity w is then used to calculate the fractional part of $\log_2 x$ from the series

$$\log_2 w = -1 + \sum_{i=1}^{39} a_i \, 2^{-i}$$ where a_i is either zero or one.

The quantities a_i are determined from the recurrence relation

$$P_0 = x$$

$$P_{i+1} = P_i^2 \text{ if } P_i^2 \geq 1/2$$

$$= 2P_i^2 \text{ if } P_i^2 < 1/2.$$

The a_i are found one at a time by the relations

$$a_{i+1} = 1 \text{ if } P_i^2 \geq 1/2$$

$$a_{i+1} = 0 \text{ if } P_i^2 < 1/2.$$

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APPROVED BY J.P.Nash

LOCATION	ORDER	NOTES PAGE 1 S 3
	0QK(S3)	
0	40 F	Store x in location 0
	K5 F	
1	42 13L	Plant link
	41 2F	
2	50 F	
	22 5L	
3	40 F	
	L5 2F	Form "0" digit for fractional part
4	26 11L	
	00 1F	
5	F5 1F	
	40 1F	Count for integer part
6	SJ F	
	32 4L	Test for $\geq 1/2$
7	S5 F	
	26 9L	
8	50 F	
	7J F	Form P^2
9	40 F	
	L4 F	
10	36 3L	Test for $P_i^2 \geq 1/2$
	F5 2F	Form "1" digit for fractional part
11	L4 2F	
	40 2F	
12	36 8L	
	50 2F	Put fractional part in Q
13	F1 1F	Put integer part in A
	22 F	Exit