UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE L 8 - 302

TITLE: Solution of a system of linear equations by an iterative method (SADOI Only)

TYPE: Complete program

NUMBER OF WORDS: 338 (20-358) in Williams Memory

ACCURACY: Depends on the condition of the equations. For more information on this point see the description of the mathematical method.

DURATION: Depends to a large extent on the initial approximation. If it's close to the exact solution, convergence will be fairly rapid; if not, then convergence is liable to be very slow. If T is the time required to solve a system of N equations in N unknown, then $T \simeq .013N^2$ minutes. For example, when N = 35, T = 16. This estimate of running time is to be regarded as an approximation only!

PARAMETERS:

S6 - 00F 00nF where n = number of equations

S9 - 00F 00kF where $1 \leq k \leq 12$ is the number of decimal places desired in the output.

RESTRICTIONS: A must be symmetric and positive definite. In addition

$$\sum_{j=0}^{N-1} |a_{ij}| < 1 \text{ for } i = 0, \ldots N - 1.$$

This implies that the largest eigenvalue of A < 1. On the data tape, to be described below, the user will have to specify a and b where a is the lower bound for the magnitude of the eigenvalues of A and b is an upper bound. 0 cannot be taken as a lower bound for the eigenvalues of A. Finally

$N \leq 100$.

STOPS: If there has been a drum failure there will be an FF stop at R.H. $(0F0)_{16}$ otherwise, when the problem is done, there will be an OF stop at $(0S9)_{16}$.

METHOD OF USE: Suppose the set of equations to be solved is $A\vec{x} = \vec{y}$ where A is an M x M positive definite symmetric matrix and $\vec{y}$ a vector with M components. Let a denote the magnitude of the smallest eigenvalue or a lower bound for the magnitude of the smallest eigenvalue and let b denote the magnitude of the largest eigenvalue or an upper bound for the magnitude of the largest eigenvalue of A. Library routines M 26 or M 20 may be used to determine these bounds.

It is essential that

$$0 < a < b < 1.$$

The data tape is then prepared as follows:

a

b

s_0 this is a scaling factor, to be discussed below

N

a_{11}, a_{12}, . . . a_{1M} } First row of matrix,

N each row of matrix must

a_{21}, a_{22}, . . . a_{2M} } be followed by the sexadecimal character N.

N

. . .

a_{M1}, . . . a_{MM} } Last row of matrix,

N

Followed by:

$s_0 y_1$
.
.
.
$s_0 y_M$
N

where $\vec{y} = (y_1 \ldots y_M)$

Followed finally by:

$s_0 \eta_{01}$
.
.
.
$s_0 \eta_{0M}$
N

where $\vec{\eta}_0 = (\eta_{01}, \ldots \eta_{0M})$ is the initial approximation

All the above numbers are signed decimal fractions which are read in via N 12.

SCALING:

s_0 will usually be 10^{-1} or 10^{-2} etc. but other scale factors such as 2^{-1}, 2^{-2}, etc. may also be used. The scale factor should be chosen so that the matrix s_0 A satisfies the restrictions noted above and the coordinate elements of $s_0\vec{y}$ and $s_0 \vec{\eta}_0$ satisfy the following inequalities (in order to prevent overflow):

1) $s_0 \sum_{j=1}^{M} |\eta_{0j}| < 1/2$

2) $s_0 \sum_{j=1}^{M} |y_j| < 1/2$

The routine automatically performs any additional scaling. The first number punched on the output tape is s_k, the final scaling factor, followed by $s_k\vec{x}$, the scaled solution vector.

BRIEF DESCRIPTION OF MATHEMATICAL METHOD AND CONVERGENCE CRITERION:

Denote the system of equation to be solved by

(1) $A\vec{x} = \vec{y}$

It is assumed throughout the following discussion that A is symmetric, positive definite, and has all eigenvalues less than 1. Now if one wants to solve this system of equations by an iterative method then, as has been pointed out by von Neumann [2], a large number of such schemes are described by the equations:

(2) $\vec{\xi}_{k+1} = G_k \vec{\xi}_k + H_k \vec{y}$

where $\vec{\xi}_k$ denotes the kth iterate and G_k and H_k are matrices of the same order as A. The G_k and H_k are not independent of one another; they must satisfy the following two conditions:

(3) $G_k + H_k A = I$ (I $\equiv$ identity matrix)

(4) $\det |H_k| \neq 0$

The reason for this is the following: suppose $\vec{\xi}_k$ is the solution i.e. $\vec{\xi}_k = \vec{x}$ then obviously $\vec{\xi}_{k+1} = \vec{\xi}_k$ which means $\vec{x} = (G_k + H_k A)\vec{x}$, all $\vec{x}$, hence (3) follows. Similar considerations lead to condition 4. For further details consult Golub [1] pp. 7-8. Since $\vec{x} = (G_k + H_k A)\vec{x}$ then upon subtracting (2):

(6) $\vec{x} - \vec{\xi}_{k+1} = G_k(\vec{x} - \vec{\xi}_k)$ (since $A\vec{x} = \vec{y}$).

Therefore

(7) $(\vec{x} - \vec{\xi}_k) = \prod_{j=0}^{k-1} G_j(\vec{x} - \vec{\xi}_0)$.

In many cases $G_k = G$(all k) so (7) becomes

(8) $\vec{x} - \vec{\xi}_k = G^k(\vec{x} - \vec{\xi}_0)$.

It is now easily seen that if the eigenvalues of G are all less than 1 in magnitude $\vec{x} - \vec{\xi}_k \to 0$ independently of $\vec{\xi}_0$. However, if $\vec{\xi}_0$ is a poor approximation or if the largest eigenvalue of G is close to 1 the convergence may be too slow. In [2] von Neumann considers replacing the original sequence $\vec{\xi}_0, \vec{\xi}_1,$ by a sequence of averages $\vec{\eta}_0, \vec{\eta}_1, \ldots$ where

$$(9) \quad \vec{\eta}_k = \sum_{\ell=0}^{k} a_k \quad \vec{\xi}_\ell$$

and

$$(10) \quad \sum_{\ell=0}^{k} a_{k\ell} = 1.$$

Hence

$$(11) \quad \vec{x} - \vec{\eta}_k = \sum_{\ell=0}^{k} a_{k\ell}(\vec{x} - \vec{\xi}_\ell) = \sum_{\ell=0}^{k} a_{k\ell} G^{\ell} (\vec{x} - \vec{\xi}_0) = \left(\sum_{\ell=0}^{k} a_{k\ell} G^{\ell}\right) (\vec{x} - \vec{\xi}_0).$$

That is, after k iterations of this process the initial error is multiplied by a matrix polynomial so that

$$(12) \quad \vec{x} - \vec{\xi}_k = P_k(G)\ (\vec{x} - \vec{\xi}_0) \text{ where } P_k(G) = \sum_{\ell=0}^{k} a_{k\ell} G^{\ell}$$

If $\vec{x} - \vec{\eta}_0$ is expanded in terms of the eigenvectors of G(if G is symmetric, this can always be done) then the following inequality results:

$$(13) \quad ||\vec{x} - \vec{\eta}_k|| \leq \max_{1 \leq i \leq N} |P_k(\lambda_i)| \cdot ||\vec{x} - \vec{\eta}_0||$$

where λ_i are the eignevalues of G. Now for arbitrary $\vec{\eta}_0$ $||\vec{x} - \vec{\eta}_k||$ is to be as small as possible after k steps, hence max $|P_k(\lambda_i)|$ must be minimized. The answer is well known and leads to the Chebyschev iteration scheme described by Golub [1] pp. 9-15. The general expression is:

$$(14) \quad \vec{\eta}_{k+1} = 2b_{k+1}(G\vec{\eta}_k + H\vec{y} - \vec{\eta}_{k-1}) + \vec{\eta}_{k-1} \quad (k = 1,2, \ldots)$$

$$b_{k+1} = \frac{1}{2-\bar{\lambda}^2 b_k}; \quad b_1 = 1$$ and $\bar{\lambda}$ is an upper bound for the largest eigenvalue.

In the library routine described here $H_k = \alpha I$, all k, and hence $G = I - \alpha A$, where α is chosen so as to minimize the maximum eigenvalue of G. If the smallest and the largest eigenvalues of A are denoted by a and b respectively, then $\alpha = \frac{2}{a+b}$. This choice of α yields the iterative method used in this routine.

CONVERGENCE CRITERION:

As is pointed out by Golub, $||\vec{\eta}_{k+1} - \vec{\eta}_k|| < \epsilon$ does not imply $||\vec{x} - \vec{\eta}_{k+1}|| < \epsilon$. However, if $\vec{z}_k$ is defined as

$\vec{z}_k = G\vec{\eta}_k + H\vec{y} - \vec{\eta}_k = HA(x - \eta_k)$, then if $||\vec{z}_k||$ is "small" so is $||\vec{x} - \vec{\eta}_k||$. More precisely

$$\alpha^{-1} A^{-1} \vec{z}_k = \vec{x} - \vec{\eta}_k \text{ so } ||\vec{x} - \vec{\eta}_k|| \leq \frac{||A^{-1}||}{\alpha} ||\vec{z}_k||.$$

Therefore if

$$||\vec{z}_k|| \leq \frac{\alpha \epsilon}{||A^{-1}||}$$

then

$$||\vec{x} - \vec{\eta}_k|| \leq \epsilon.$$

Now

$$HA(\vec{x} - \vec{\eta}_k) = \alpha A(\vec{x} - \vec{\eta}_k) = \alpha(\vec{y} - A\vec{\eta}_k) = \alpha \vec{r}_k = \vec{z}_k.$$

Because of round off errors ϵ cannot be made arbitrarily small. In fact it might be the case that $\min_k ||z_k|| \geq \delta > 0$ for all k. Hence, if ϵ is chosen too small, the process may not converge. A lower bound for ϵ has been determined by Golub (see [1] P. 86) which can be expressed as

$$\epsilon \leq \frac{M}{(1-\rho)^2}$$

where

$$\rho = \frac{\lambda}{1+\sqrt{1-\lambda^2}}$$

$$M = 70 \cdot 2^{-39}$$

and

$$\lambda = \frac{b-a}{b+a}.$$

M is an estimate of the rounding error which occurs during iteration. It should be noted that if ρ is close to 1 (which is the case for ill conditioned matrices) ϵ becomes large. Therefore, much less accuracy is to be expected in this case.

REFERENCES:

[1] Digital Computer Laboratory, University of Illinois, Urbana, Illinois, report No. 85. (1959)

[2] Los Alamos Scientific Laboratory report LA - 2165 (1958) pp. 46-68.

DATE July 19, 1960

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aj

LOCATION		ORDER		NOTES PAGE 1 L 8
		00 10K		
		26 1000N		
0		L5 6F		361 + 2N
		00 1F		
1		L4 10L		
		42 3F		(S3)
2		50 6F		
		75 6F		
3		L5 6F		$N^2 + N + 2560$
		S4 F		
4		L4 11L		
		40 4F		(S 4)
5		L5 6F		
		00 1F		3N + 1 + 360
6		L4 6F		
		L4 10L		
7		42 5F		(S 5)
		L5 12L		
8		42 7F		
		L5 13L		
9		42 8F		
		26 999F		
10		00 F		
		00 361F		
11		00 F		
		00 2560F		
12		00 F		
		00 202F		
13		00 F		
		00 251F		
		26 10N		
		00 20K		

LOCATION		ORDER		NOTES PAGE 2 L 8
		00K(L9)		
0		40 (506)		Store scale parameter
		L5 0F		
1		10 1F		
		40 0F		a/2
2		L5 1F		
		10 1F		
3		40 1F		b/2
		L4 0F		
4		40 (507)		a/2 + b/2
		L5 1F		
5		L0 0F		
		50 (508)		Clear Q
6		66 (507)		
		S5 F		
7		40 (509)		λ = b-a/b+a
		75 (509)		
8		10 1F		
		40 (510)		$\lambda^2/2$
9		L5 (501)		
		L4 (505)		
10		40 (503)		
		L5 (500)		
11		L4 (505)		
		40 (502)		
12		L4 (511)		
		40 (504)		
13		50 (509)		
		71 (509)		
14		L4 (511)		
		40 1F		
15		S5 F		
		40 0F		
16	(0)	00 1F		
		50 (0)		

LOCATION		ORDER		NOTES PAGE 3 L 8
17		22 (R1)		$\sqrt{1 - \lambda^2}$
		10 1F		
18		40 OF		
		LJ OF		
19		40 OF		
		50 (508)		
20		L5 (509)		
		10 1F		
21		66 OF		$\rho = \lambda/1 + \sqrt{1-\lambda^2}$
		S5 F		
22		L4 (511)		
		40 OF		
23		50 OF		
		7J OF		$(1 - \rho)^2$
24		40 OF		
		L5 (512)		
25		50 (508)		
		66 OF		
26		S7 F		
		40 (513)		Compute end constant
27	(25)	49 OF	from 108	
		L1 OF		
28		40 (514)		
		41 (523)		
29	(24)	22 (1)		
		L5 (514)	from 90, 99	
30		00 1F		
		36 (2)		Is $-b_k = -1/2$?
31		89 1F		
		40 (514)		
32	(2)	50 (514)	from 30	
		79 (510)		
33		L4 (511)		
		40 OF		
34		50 (508)		
		LJ (508)		

LOCATION		ORDER	NOTES	PAGE 4 L 8
35		66 OF		
		S5 F		
36	(1)	40 (514)		Compute - b_{k+1}
		41 (517)	from 29	
37		41 (515)		
		41 (516)		
38		L5 (500)		
		40 (3)		
39		40 (19)		
		L5 (501)		
40		40 (4)		
		L5 (518)		
41		40 (5)		
		42 (6)		
42	(17)	L5 (520)	from 82	
		42 (7)		
43		42 (8)		
		42 (9)		
44		46 (6)		
		42 (11)		
45	(3)	85 11F		
		40 S4	by 38,47; from 50	Read in 128 components of $\Delta \vec{\eta}_{k-1}$
46	(7)	32 (7)		
		40 S5	by 42,49	
47		F5 (3)		
		40 (3)		
48		L0 (502)		
		36 (18)		
49		F5 (7)		
		42 (7)		
50		L0 (521)		
		36 (3)		
51	(18)	L5 (6)	from 48,75	
		40 1F		
52		L5 (517)		
		40 OF		

LOCATION		ORDER	NOTES	PAGE 5 L 8
53	(10)	L5 (506)		
		50 (10)		
54		26 S7		Jump to residual
		40 OF		
55		L7 OF		
		L2 (507)		
56		36 (100)		Rescale
		L5 OF		
57		66 (507)		
		S5 F		
58	(8)	40 OF		
		L4 S5	by 43, 74	
59		40 1F		
		19 1F		
60		50 1F		
		70 (514)		
61	(9)	00 1F		
		L0 S5	by 43, 74	
62	(6)	40 S5	$\Delta\vec{\eta}_k$; by 44, 73	
		L4 S3	by 41, 73	
63		40 1F		
		LL 1F		
64		36 (12)		
		26 (100)		Rescale
65	(12)	L5 (515)	from 64	
		32 (13)		
66	(13)	26 (14)		
		50 OF	from 65	
67		L5 (516)		
		74 OF		
68		L4 (515)		
		40 (515)		
69		S5 F		
		40 (516)		
70	(14)	F5 (517)	from 66	
		40 (517)		

LOCATION		ORDER	NOTES	PAGE 6 L 8
71		L0 (505)		
		36 (15)		
72		L5 (6)		
		L4 (519)		
73		40 (6)		
		F5 (8)		
74		42 (8)		
		42 (9)		
75		L0 (524)		
		36 (18)		
76	(11)	00 1F	from 71	
	(15)	L5 S5	by 44,80; from 81	
77	(4)	86 11F		
		00 S4	by 40,78	Record 128 components of $\Delta\vec{\eta}_k$
78		F5 (4)		
		40 (4)		
79		L0 (503)		
		36 (19)		
80		F5 (11)		
		42 (11)		
81		L0 (525)		
		32 (11)		
82		26 (17)		
		00 F		
83	(19)	85 11F	from 79,88	
		00 S4	by 39, 87	
84	(5)	L4 S3		
		40 S3	by 41,86	
85		L5 (5)		
		L4 (519)		
86		40 (5)		
		F5 (19)		
87		40 (19)		
		L0 (504)		
88		36 (19)		
		L5 (516)		

LOCATION		ORDER		NOTES PAGE 7 L 8
89		40 OF		
		L5 (515)		
90		32 (20)		
		22 (24)		
91	(20)	00 1F		
		50 (20)	from 90	
92		26 (R1)		
		40 OF		
93		L3 (523)		
		36 (22)		is $\vec{r}_k = 0$?
94		L5 (523)		
		L0 (513)		
95		36 (23)		
		L5 OF		$\|\|\alpha \vec{r}_k\|\|$ - end constant < 0?
96		L0 (523)		
		36 S8		← C - θ! when done
97	(23)	L5 (523)	from 95	
		40 (522)		
98	(22)	L5 OF	from 93	
		40 (523)		
99		22 (24)		
		00 F		$\|\|\alpha \vec{r}_{k+1}\|\| = \|\|\alpha \vec{r}_k\|\|$ in acc.
100	(100)	L5 (526)	from 56,64	
		40 (101)		
101	(103)	41 OF		
		50 (102)	from 106	
102	(101)	7J S3		
		40 S3	by 100, 104	
103		L5 (101)		
		L4 (519)		
104		40 (101)		
		F5 OF		
105		40 OF		
		L0 (505)		
106		36 (104)		
		22 (103)		

LOCATION		ORDER		NOTES PAGE 8 L 8
107	(104)	50 (102)	from 106	
		7J (506)		
108		40 (506)		
		26 (25)		
109	(102)	40 F		
		00 F		
110	(500)	85 11F		
		00 S4		
111	(501)	86 11F		
		00 S4		
112	(502)	85 11F		
		00 F	by 11	85 11F 00 NS4
113	(503)	86 11F		
		00 F	by 10	86 11F 00 NS4
114	(504)	05 11F		
		00 F	by 12	05 11F 00 NS4
115	(505)	00 F		
		00 S6		00 F 00 NF
116	(506)	00 F		
		00 F	by 0, 108	Scaling factor
117	(507)	00 F		
		00 F	by 4	$a/2 + b/2 = 1/\alpha$
118	(508)	00 F		
		00 F		zero
119	(509)	00 F		
		00 F	by 7; λ	
120	(510)	00 F		
		00 F	by 8	$\lambda^2/2$
121	(511)	80 F		
		00 F		-1
122	(512)	00 F		
		00 70F		
123	(513)	00 F		
		00 F	by 26	End constant

LOCATION		ORDER	NOTES	PAGE 9 L 8
124	(514)	00 F		
		00 F	by 28,31,36	$-b_k$
125	(515)	00 F		
		00 F	by 68	$\|\|\alpha \vec{r}_{k+1}\|\|^2_M$
126	(516)	00 F		
		00 F	by 69	$\|\|\alpha \vec{r}_{k+1}\|\|^2_L$
127	(517)	00 F		
		00 F	by 36,70	Equation counter
128	(518)	L4 S3		
		40 S3		
129	(519)	00 1F		
		00 1F		
130	(520)	00 S5		
		00 S5		
131	(521)	S2 (7)		
		40 128S5		
132	(522)	00 F		
		00 F	by 99	$\|\|\alpha r_{k-1}\|\|$
133	(523)	00 F		
		00 F	by 28, 98	$\|\|\alpha r_k\|\|$
134	(524)	NO F		
		L4 128S5		
135	(525)	80 1F		
		L5 128S5		
136	(526)	7J S3		
		40 S3		
		00K(R1)		

LOCATION		ORDER		NOTES PAGE 10 L 8
		00 K		
0	(A)	50 33L		Read in a, b, s_0
		50 L		
1		26 (N12)		
		L5 30L		
2		42 4L		
		42 9L		
3		50 360F		Read in one row of matrix and transfer
		50 3L		it to drum
4		26 (N12)		
		L5 F		
5		86 11F		
		00 2560F		
6		F5 5L		
		40 5L		
7		F5 4L		
		40 4L		
8		L0 31L		
		32 4L		
9		41 29L		Check sum
		L5 F		
10		L6 29L		Computation
		40 29L		
11		F5 9L		
		40 9L		Form $x_i + \lvert S_{i-1} \rvert = S_i$
12		L0 32L		
		32 13L		$S_{-1} = 0$
13		22 9L		
		L5 5L		
14		40 15L		Store $-S_n - 2^{-39}$ = CKS
		F1 29L		
15		00 F		on drum just after last elt. of row
		00 F		
16		F5 5L		
		40 5L		

LOCATION		ORDER		NOTES PAGE 11 L 8
17		F5 (C)		
		40 (C)		
18		L0 (N)		
		36 20L		
19		22 1L		
		0F F		
20		50 361S6		Read in s_0y and initial approximation
		50 20L		
21		26 (N12)		
		26 22L		
22		50 S3		
		50 22L		
23		26 (N12)		
		L5 33L		
24		40 F		
		L5 34L		
25		40 1F		
		L5 35L		
26		40 (102)		
		26 20F		Jump to iteration routine
27	(C)	00 F		
		00 F		
28	(N)	00 F		
		00 S6		
29		00 F		
		00 F		
30		00 F		
		00 360F		
31		K6 (N12)		
		L5 360S6		
32		41 29L		
		L5 360S6		
33		00 F		
		00 F		
34		00 F		
		00 F		

LOCATION		ORDER		NOTES PAGE 12 L 8
35		00 F		
		00 F		
		00 K		
0	(B)	40 46L		Store scale factor;
		K5 F		
1		42 37L		Plant link
		L5 F		
2		40 (I)		Store i
		50 (N+1)		
3		75 (I)		$i(N+1)\ 2^{-39} + 2560$
		S5 F		
4		L4 40L		
		40 7L		
5		L4 (N+1)		End constant for drum transfer loop
		40 41L		
6		L5 30(A)		
		42 8L		
7		00 F		Drum read order inserted by 4
		00 F		
8		22 8L		
		40 F		
9		F5 8L		
		40 8L		Transfer ith row of matrix, including CKS
10		F5 7L		to W.M. 360, 361, ... 360+N
		40 7L		
11		L0 41L		
		32 12L		
12		26 7L		
		41 29(A)		
13		L5 30(A)		
		42 14L		Form CKS
14		L7 29(A)		
		L4 F		
15		40 29(A)		
		F5 14L		

LOCATION		ORDER		NOTES PAGE 13 L 8
16		40 14L		
		L0 42L		
17		36 18L		
		26 14L		
18		L5 8L		
		L0 43L		
19		42 20L		
		F5 26(A)		
20		22 20L		
		L4 F		
21		40 F		
		L3 F		
22		36 23L		Old CKS = New CKS?
		FF F		$\geq$ 0 yes; < 0 No!
23		41 (MSP)		
		49 (LSP)		
24		L5 30(A)		
		42 26L		
25		L5 48L		
		46 27L		
26		L5 (LSP)		
		50 F		Compute residual,
27		74 F		
		L4 (MSP)		
28		40 (MSP)		double precision
		S5 F		
29		40 (LSP)		
		L5 27L		
30		L4 43L		
		46 27L		
31		F5 26L		
		42 26L		
32		L0 47L		
		32 33L		
33		26 26L		

LOCATION		ORDER		NOTES PAGE 14 L 8
		L5 48L		$s_k y_i = s_k\left(\frac{s_0 y_i}{s_0}\right)$
34		L4 (I)		
		42 35L		
35		50 46L		
		75 F		
36		66 35(A)		
		S5 F		
37		L0 (MSP)		
		22 F		
38	(I)	00 F		
		00 F		
39	(N+1)	00 F		
		00 1S6		
40		85 11F		
		00 2560F		
41		00 F		
		00 F		
42		L7 29(A)		
		L4 360S6		
43		00 1F		
		00 1F		
44	(MSP)	00 F		
		00 F		
45	(LSP)	00 F		
		00 F		
46		00 F		s_k stored here
		00 F		
47		L5 (LSP)		
		50 360S6		End constant
48		00 S3		
		00 361S6		

LOCATION		ORDER		NOTES PAGE 15 L 8
		00 K		
0		L5 (506)		
		26 1L		
1		50 12F		Print out final scale factor
		50 1L		
2		26 (P16)		
		L5 5L		
3		L4 (N)		
		40 11L		
4		92 135F		
		92 515F		
5		22 5L		
		L5 S3		
6		50 S9		
		50 6L		
7		26 (P16)		
		92 131F		
8		92 515F		
		F5 5L		
9		40 5L		
		L0 11L		
10		32 19(A)		
		22 5L		
11		00 F		
		00 F		
		(P16) 00K		
		(N12) 00K		
		24(A) 261N		