

UNIVERSITY OF ILLINOIS
DIGITAL COMPUTER

LIBRARY ROUTINE K 14 - 233

By Gene H. Golub

TITLE Multiple Regression Analysis with Transformations

TYPE Entire Program

METHOD OF USE

- 1) Read in master program. A sum check is performed, and if the program is read correctly the machine will stop on $24 (012)_{16}$; otherwise it will stop on an FF order.
- 2) Read in the parameter tape. The computer stops on $20 (ONL)_{16}$.
- 3) Read in the data tape.
- 4) After all data have been read into the computer, the desired information will be printed out. The computer stops on $24 (012)_{16}$.
- 5) Another problem can be begun by reading in new parameters.

CAPACITY The maximum number of non-eliminated variables is twenty-two. The total number of variables must be ≤ 50 . The number of observation is unlimited.

This program hangs up unless $s > n + 1$.

DURATION [See Note 3]

PUNCHING OF THE TAPES For each problem six parameters are necessary. They are as follows:

- (1) Let "s" be the sample size. Put sS on the tape.
- (2) Let "n" be the number of non-eliminated variables. Put nN on the tape.
- (3) Let "f" be the number of decimal places to which correlations are to be printed. Put fF on the tape.
- (4) Let " ℓ " be the number of decimal places to which covariances are to be printed. Put ℓL on the tape.
- (5) Let "k" be the number of decimal places to which standardized regression weights, their standard errors, and multiple correlations are to be printed. Put kK on the tape.

(6) Let "j" be the number of decimal places to which unstandardized regressions weights, their standard errors and standard errors of estimate are to be printed. Put jJ on the tape.

Any of the four parameters f, j, k, or *l* may be set to zero. The parameters are followed by a sequence of 1's and 0's. The 1's indicate which of the non-eliminated variables are to be considered successively independent. (e.g., if this sequence is a single 1 the first variable will be considered dependent, if it consist of 1001 the first variable and then the fourth variable will be considered dependent). 0's following the last 1 in the sequence should be omitted. If more than one variable is to be considered dependent no delays, spaces, line feeds or other characters with the fifth hole punched may be inserted between the first 1 and the last 1 in this sequence. The first character after the last 1 in the sequence must be a character with the fifth hole punched, (e.g., delay, space, line feed, etc). The portion of the input tape between this character and a character which consists of a single fifth hole on the tape will be exactly reproduced on the output tape. In this interval some identification may be punched. The identification can be a number, a name, or any combination of characters which does not include a character consisting a single fifth hole on the tape. It must, however, be followed by such a character. If no identification is desired, two fifth hole characters must be punched immediately after the last 1 in the sequence. The second of these must consist of a single fifth hole on the tape. No fifth hole character may be punched following the S, N, F, L, J, K parameters in the section where the successive dependent variables are indicated. If one is punched, characters between the fifth hole character the single fifth hole delay will be reproduced as identification.

Next, a sequence of signed integers, followed by an N, is punched on the tape. These integers take on the values 0, 1, 2, 3, 4, 5, 6. Corresponding to each integer, the following transformations are made:

n	transformation on x
+0	eliminate variable
+1	x
+2	x ²
+3	x ³
+4	$\sqrt{x}, x \geq 0$
+5	$\log x/10, x \geq 10^{-10}$
+6	$\frac{1}{x} \text{ arc sin } x$

The number of digits in this sequence must be equal to the total number of variables.

Each element of an observation vector must be punched as a sign followed by up to 12 decimal digits. The character N must be punched after each row. If an F follows a row, then the machine will stop, and another part of the data tape may be placed in the reader.

SCALING OF THE DATA

Each observation must be scaled so that it is less than one in absolute value before it can be punched. Each element should be scaled by the same value if the constant term is to be easily interpreted. If each element is scaled by a constant c , then the correlation coefficients, multiple correlation, regression coefficients and standard error of the regression coefficients will be unaffected. The means, standard deviations, standard estimate of error and constant term will all be scaled by c . The variances and covariances will be scaled by c^2 .

THE PRINT OUT

This is best illustrated by a typical example. For a problem with four variables, the print out could appear as follows on the next page. The means and standard deviations are always printed to nine places.

r_{11} →	+1.0000000	+0.01386883	
r_{21} →	+0.1649341	+0.00459466	
r_{22} →	+1.0000000	+0.05595594	
Correlations →	+0.8340721	+0.00510478	← Covariances
	+0.5308620	+0.00652617	
	+1.0000000	+0.00270089	
r_{41} →	+0.3013304	+0.00204625	
r_{42} →	-0.4067406	-.00554800	
r_{43} →	+0.2975063	+0.00089155	
r_{44} →	+1.0000000	+0.00332500	
Means →	+0.214150000	+0.117765986	← Standard deviations
	+0.549600000	+0.236550079	
	+0.106490000	+0.051970115	
	+0.105000000	+0.057662813	

	+0.043813563	← Standard error of estimate.		
	+0.928217019	← Multiple correlation		
Dependent Variable →	-1.00000000	+0.00000000	-1.00000000	+0.00000000
	-0.33097420	+0.076293334	-0.66480972	+0.153246236
	+2.92808434	+0.332283964	+1.29216327	+0.146636873
	-0.72196344	+0.277815345	-0.35350141	+0.136029212
Constant term →	+0.16004788			
	↓	↓	↓	↓
	Unstandardized	Standard	Standardized	Standard
	Regression Weights	Errors	Regressions Weights	Errors

SUM CHECK

Throughout this program, there are a number of checks on the tapes and magnetic drum storage. They are as follows:

- 1.) If the master tape has been read in incorrectly, the machine will stop on an FF 011 order at $(378)_{16}$.
- 2.) If the data tape does not have the same number of variables in each row, the machine stops on an FF 012 order at $(0J4)_{16}$.

3.) If a drum error has been made, the machine will stop on an FF 013 order at (06N)₁₆.

MATHEMATICAL MODEL

Let x_{ij} be the j th observation on the i th variable. As each observation vector is read in,

$$\sum_{j=1}^p \frac{x_{ij} x_{kj}}{s} \quad \text{and} \quad \sum_{j=1}^p \frac{x_{ij}}{s} \quad \begin{array}{l} p = 1, 2, \dots, s \\ k \leq i \\ i = 1, 2, \dots, n \\ k = 1, \dots, n \end{array}$$

are computed. After all the observations have been read in, the correlation matrix is calculated and then inverted using a modified version of M 14. The formulas used for calculation are as follows:

The mean:
$$\bar{x}_i = \frac{\sum_{j=1}^s x_{ij}}{s}$$

The Covariance:
$$c_{ik} = \sum_{j=1}^s \frac{x_{ij} x_{kj}}{s} - \bar{x}_i \bar{x}_k$$

The Correlation Coefficient:
$$r_{ik} = \frac{c_{ik}}{\sqrt{c_{ii} c_{kk}}}$$

The Standardized Regression Weight:

$$b_{ik}^* = \frac{-r_{ik}}{r_{ii}}, \quad k = 1, \dots, n$$

where r^{ik} is the (i,k) element of the inverse of the correlation matrix.

The Standard Error of the Standardized Regression Weights:

$$S_{b_{ik}^*} = \sqrt{\frac{r_{ii} r_{kk} - (r_{ik})^2}{(s - n) (r_{ii})^2}}$$

The Multiple Correlation Coefficient:

$$R_i = \sqrt{1 - \frac{1}{r_{ii}}}$$

The Standard Error of Estimate:

$$S_i = \sqrt{c_{ii} (1 - R_i^2)}$$

The Unstandardized Regression Weights:

$$b_{ik} = \frac{c_{ii}}{c_{kk}} b_{ik}^*$$

The Standard Error of the Unstandardized Regression Weights:

$$S_{b_{ik}} = \sqrt{\frac{c_{ii}}{c_{kk}}} S_{b_{ik}}^*$$

NOTE 1

Beginning at drum location 2560, the subroutines necessary for this program are stored.

NOTE 2

Beginning at drum location 3584, the means, standard deviations, correlation matrix and its inverse are stored.

NOTE 3

A TABLE OF TYPICAL TIMES

Number of Variables	Read in 100 Observations of 8 digits	Print Correlations and Covariance Matrix to 8 Places	Invert Correlation Matrix	Print Standardized and Unstandardized Regression Weights and their Standard Error to 8 Places
5	45 sec.	10 sec.	4 sec.	6 sec.
10	95 sec.	32 sec.	10 sec.	12 sec.
15	150 sec.	68 sec.	18 sec.	18 sec.
20	218 sec.	120 sec.	35 sec.	23 sec.

In addition, it requires about 35 seconds to read the master tape into the machine.

Rt: 7/1/60

DATE March 20, 1957

PROGRAMMED BY S. G. Galt

APPROVED BY D. E. Muller

LOCATION	ORDER	NOTES	PAGE 1	K 14
Decimal Order Input- X-1				
	00 3K			
3	00 F	Location of data, means and matrix of cross-products		
	00 500F			
4	00 F	Location of Library Routine N 12		
	00 960F			
5	00 F	Location of List of Constants		
	00 58F			
6	00 F	Location of Library Routine R-1		
	00 116F			
7	00 F			
	00 3584F	First location of Data Stored on Drum		
8	00 F	Location of Diagonal Elements of Inverse and Scaling Factors.		
	00 982F			
9	00 F	Location of Library Routine Y-1		
	00 76F			
10	00 F	Location of Library Routine P-1 (modified)		
	00 125F			
11	00 F			
	00 2560F	First Location of Subroutines on Drum		
12	00 F			
	00 700F			
13	00 F			
	00 400F	Location of directory		
14	00 F			
	00 275F	Location of transformation routine		
	00 76K			
	Library Routine Y-1	Transfer Blocks of Words from the Memory to the Drum or from the Drum to the Memory Change FF order in Y-1 here to read FF 19F		
	Subroutine A			
	This subroutine clears the memory, inputs the parameters, and translates the identification.			
	00 200K			
0	K5 F			
	42 40L	Plant link		

LOCATION	ORDER		NOTES	PAGE 2
1	L5 43L 40 2L			
2	00 11F 41 ()F	from 4 by 1,3	Clear memory	
3	F5 2L 40 2L			
4	L0 41L 36 2L			
5	92 135F 41 7F	from 13		
6	81 4F L0 13S5	from 10		
7	32 10L L4 13S5		Read in parameters and convert	
8	50 7F 74 13S5			
9	S5 F 40 7F			
10	26 6L 42 11L	from 7		
11	L5 7F 40 ()F	by 10		
12	F5 8F 40 8F			
13	L0 42L 32 5L			
14	40 8F 81 4F			
15	F0 5S5 32 17L	from 19	Read in sequence of 1's and 0's to determine dependent variables	
16	L5 9F L6 8F			
17	40 9F L5 8F	from 15		
18	10 1F 40 8F			

LOCATION	ORDER		NOTES	PAGE 3
19	91 4F 36 15L			
20	26 25L L5 7F	from 26		
21	32 22L 50 24L		read in identification and print out	
22	26 23L 50 23L	from 21		
23	04 6F 42 24L			
24	02 1F 92 ()F	by 23		
25	91 4F 40 7F	from 20		
26	L7 7F 32 20L			
27	92 707F 92 131F			
28	92 515F L5 2F			
29	42 0S5 42 6S5		Set parameters	
30	42 16S5 00 20F			
31	46 S5 L5 S5			
32	L4 1S5 40 2S5			
33	L4 S5 40 3S5			
34	L5 4F 42 8S5			
35	L5 5F 42 9S5			

LOCATION	ORDER	NOTES	PAGE 4
36	L5 OF 42 10S5		
37	L5 3F 42 11S5		
38	L5 1F 40 15S5		
39	L5 9F 40 14S5		
40	00 1F 22 ()F	by 0	
41	80 11F 41 1034F		
42	80 F 00 6F		
43	00 11F 41 S3 00 800K		
0	J0 200F 50 L		
1	26 S9 00 SS	Store subroutine A on drum	
2	00 44F L5 OF	[This is an interlude which is later overwritten.]	
3	40 4L 26 999F 26 800N		
Subroutine B			
This subroutine computes the means and sum of cross-products divided by s.			
0	00 200K S5 50F 40 0L		
1	52 SJ 50 1L	Read in integers determining transformations	
2	26 S4 L5 21S4		

LOCATION	ORDER		NOTES	PAGE 5	K 14
3	40 56L L0 6L				
4	10 20F 42 53SF				
5	50 L 24 7L				
6	40 SJ L5 21S4				
7	50 8L 26 999F				
8	00 F 00 7L				
9	00 F 26 7L 26 1N				
0	L5 15S5 L4 5S5				
1	40 4F K5 F		Plant link		
2	42 48L 19 38F				
3	66 15S5 S5 F		1/s		
4	40 5F L5 49L				
5	40 6F L4 7L				
6	46 6F 41 9F				
7	50 49F 50 7L	from 39	Read in and transform observation row		
8	26 SF L0 12S5				
9	30 9L L5 21S4				

LOCATION	ORDER		NOTES
10	L0 6F 40 OF		
11	L3 OF 32 12L		
12	FF 18F L5 1S5		Have n observations been read in?
13	42 15L L5 2S5		
14	42 16L 46 16L		
15	50 5F 7J ()F	from 21 by 13,17	
16	L4 ()F 40 ()F	by 14,19 by 14,19	$\sum_{k=1}^p \frac{x_{ik}}{s}$
17	F5 15L 42 15L		p = 1, 2, ... s
18	L5 16L L4 4S5		
19	40 16L F5 OF		
20	40 OF L0 6S5		
21	36 15L F5 5S5		
22	40 2F L5 1S5		
23	42 26L L5 3S5		
24	42 29L 46 29L		
25	L5 1S5 42 28L	from 36	
26	41 1F 50 ()F	by 23,35;from 34	

LOCATION	ORDER		NOTES	PAGE 7
27	7J 5F 40 OF			
28	50 OF 7J ()F	by 25,30	$\sum_{k=1}^p \frac{x_{ik} x_{jk}}{s}$	
29	L4 ()F 40 ()F	by 24,32 by 24,32		
30	F5 28L 42 28L			p = 1, 2, ... s
31	L5 29L L4 4S5			
32	40 29L F5 1F			
33	40 1F L0 2F			
34	32 26L F5 26L			
35	42 26L F5 2F			
36	40 2F L5 1F			
37	L0 6S5 36 25L			
38	F5 9F 40 9F			
39	L0 4F 36 7L			
40	41 OF L5 3S5			
41	42 42L L5 1S5			
42	42 43L L5 ()F	by 41,46;from 48		
43	22 43L 40 ()F	by 42,44		
44	F5 43L 42 43L		Store diagonal elements consecutively	

Have s observations been read in?

LOCATION	ORDER	NOTES	PAGE 8	K 14
45	F5 42L F4 OF			
46	42 42L F5 OF			
47	40 OF L0 6S5			
48	32 42L 22 ()F	by 2		
49	40 S3 L5 21S4			
50	00 F 00 52L			
51	50 50L 26 999F			
52	00 1F 26 51L 26 1N			
Subroutine C				
This subroutine computes the standard deviations				
0	K5 F 42 14L		Plant link	
1	L5 2S5 42 5L			
2	46 5L L5 1S5			
3	42 8L 46 6L			
4	41 OF 41 3F			
5	50 ()F	by 1,10; from 13		
6	79 ()F 14 ()F 32 7L	by 1,10 by 3,11		

$$\frac{\sum_{k=1}^s x_{ik}^2}{s} = \left(\frac{\sum x_{ik}}{s} \right)^2$$

LOCATION	ORDER		NOTES	PAGE 9	K 14
7	23 8L		If the variance is negative, replace		
	50 7L	from 6	by zero.		
8	26 S6		Compute standard deviation		
	40 ()F	by 3,11;			
9	L5 5L	from 7			
	L5 4S5				
10	40 5L				
	L0 S5				
11	46 6L				
	42 8L				
12	F5 3F				
	40 3F				
13	L0 6S5				
	36 5L				
14	00 1F				
	22 ()F	by 0			
Routine H					
Read in data and perform transformations					
0	00 275K				
	50 50SJ				
	50 0L				
1	26 S4				
	40 13F				
2	41 10F		Read in data		
	L5 17L				
3	42 9L				
	L5 49L				
4	42 7L				
	00 1F				
5	L5 21L				
	42 6L				
6	L5 16L	from 14			
	L4 SJ	by 5,12			
7	42 8L				
	L5 50SJ	by 4,11			

LOCATION	ORDER		NOTES	PAGE 10
8	40 1F			
	26 ()F	by 7	Determine transformation	
9	7J 1F	from 8,42		
	40 S3	by 3,10;from 17,44,49,35,36		
10	F5 9L			
	42 9L			
11	F5 7L	from 16		
	42 7L			
12	F5 6L			
	42 6L			
13	F5 10F			
	40 10F			
14	L0 53L			
	36 6L			
15	L5 13F			
	22 215F		Jump to subroutine B	
16	26 11L		eliminate x	
	00 16L			
17	22 9L		x	
	00 S3			
18	50 1F		x^2	
	26 9L			
19	50 1F		x^3	
	26 41L			
20	41 0F		\sqrt{x}	
	22 43L			
21	22 45L		$\log_{10} x/10$	
	00 SJ			
22	40 3F		$\frac{1}{\pi} \arcsin x$	
	50 3F			
23	7J 3F		$x^2 \rightarrow 12$	
	40 12F			
24	L4 54L		80 F00F	
	40 1F			
25	41 0F			
	L1 1F			

LOCATION	ORDER	NOTES		PAGE 11
26	32 26L			K 14
	50 26L			
27	26 S6		$\sqrt{1-x^2}$	
	50 54L			
28	LJ 12F			
	36 37L			
29	L5 2F			
	66 3F			
30	S7 F			
	50 30L		arc tan	
31	26 366F			
	40 OF			
32	50 OF			
	79 51L		$1/\pi$	
33	L4 50L		$1/2$	
	40 1F			
34	L5 3F			
	36 36L			
35	L1 1F			
	22 9L			
36	L5 1F	from 34		
	22 9L			
37	L5 3F	from 28		
	66 2F			
38	S5 F			
	50 38L			
39	26 366F		arc tan	
	40 1F			
40	50 51L			
	26 9L			
41	7J 1F	from 19		
	40 OF			
42	50 OF		x^3	
	26 9L			
43	00 1F			
	50 43L	from 20		

LOCATION	ORDER		NOTES	PAGE 12
44	22 S6		\sqrt{x}	
	22 9L			
45	32 45L			
	50 45L			
46	26 330F		$\log_e x$	
	40 0F		32	
47	19 5F			
	50 0F			
48	74 52L		$\frac{1}{10} \log_{10} x$	
	00 5F			
49	22 9L			
	00 50SJ			
50	40 F			
	00 F			
51	00 F		$= \frac{1}{\pi}$	
	00 3183 0988 6184J			
52	00 F			
	00 0434 2944 8195J			
53	80 F			
	00 ()F	by word 4, Routine B		
54	80 F			
	00 F			
	00 330K			
	Routine S-5 - 1/32 Natural Logarithm			
	00 366K			
	Routine T-4 - New arc Tan			
	00 810K			
0	J0 200F		Store subroutines	
	50 L		B, C, and H on drum.	
1	26 S9			
	00 700SS			
2	00 191F		[This is an interlude which is	
	L5 0F		later overwritten.]	
3	40 4L			
	26 999F			

LOCATION	ORDER	NOTES	PAGE 13
	26 810N		
	Subroutine D		
	This subroutine computes and prints correlations, covariances means, and standard deviations.		
	00 200K		
0	K5 29L		
	42 56L		Plant link
1	L3 8S5		
	36 6L		Should correlations be printed?
2	50 8S5		
	75 13S5		
3	K5 F		
	L4 4S5		
4	00 20F		
	46 28L		
5	L5 26L		
	22 6L		
6	L5 0L	from 1	
	46 27L	from 5	
7	L3 9S5		
	32 11L		Should covariances be printed?
8	50 9S5		
	75 13S5		
9	K5 F		
	00 20F		
10	46 31L		
	L5 32L		
11	26 12L		
	L5 59L	from 7	
12	42 29L	from 11	
	F5 5S5		
13	40 5F		
	L5 3S5		
14	42 20L		
	42 26L		

LOCATION	ORDER		NOTES	PAGE 14	K 14
15	L5 2S5 46 20L				
16	L5 1S5 46 22L				
17	41 6F L5 2S5	from 44			
18	42 19L L5 1S5				
19	42 21L L5 (0)F	by 18, 34; from 39			
20	79 ()F 14 ()F	by 15, 42 by 14, 37			
21	40 3F 50 ()F	by 19, 35			
22	7J ()F 40 OF	by 16, 41			
23	L3 OF 32 27L				
24	50 3F 75 57L		Compute correlations coefficient		
25	66 OF S5 F				
26	50 28L 40 ()F	by 14, 36			
27	22 ()F 23 26L	by 6 from 23			
28	52 ()F 50 28L	by 4	Print correlations		
29	26 SK 26 ()F	by 12			
30	92 963F L5 3F				
31	52 ()F 50 31L	by 10	Print covariances		
32	26 SK 50 30L		Waste		

LOCATION	ORDER	NOTES	PAGE 15	K 14
33	92 131F			
	92 515F			
34	F5 19L			
	42 19L			
35	F5 21L			
	42 21L			
36	F5 26L			
	42 26L			
37	42 20L			
	F5 6F			
38	40 6F			
	L0 5F			
39	32 19L			
	92 131F			
40	L5 22L			
	L4 4S5			
41	46 22L			
	L4 S5			
42	46 20L			
	F5 5F			
43	40 5F			
	L5 6F			
44	L0 6S5			
	36 17L			
45	L5 2S5			
	42 47L			
46	L5 1S5			
	42 49L			
47	41 3F			
	L5 ()F	by 45, 52;	from 56	
48	52 91F		Print means	
	50 48L			
49	26 SK			
	L5 ()F	by 46, 53		

LOCATION	ORDER	NOTES	PAGE 16
50	52 91F 50 50L		
51	26 SK F5 47L	Print standard deviations	
52	42 47L F5 49L		
53	42 49L 92 131F		
54	92 515F F5 3F		
55	40 3F L0 6S5		
56	32 47L 22 ()F	by 0	
57	00 F 00 1000 000 0000J		
58	L3 8S5 36 34L		
59	26 33L 00 58L 00 820K		
0	J0 200F 50 L		
1	26 S9 00 179SS	Store subroutine	
2	00 60F L5 0F	D on drum.	
3	40 4L 26 999F 26 820N	[This is an interlude which is later overwritten.]	
Subroutine E			
This subroutine stores the standard deviations, means and entire correlation matrix on the drum.			

LOCATION	ORDER		NOTES
0	00 200K K5 F 42 28L		Plant link
1	L5 S5 46 22L		
2	L4 S5 46 7L		
3	L4 4S5 10 20F		
4	L4 6L 40 21L		
5	J0 S3 50 5L		Store standard deviations and means on drum
6	26 S9 00 S7		
7	00 ()F L5 3S5	by 2	
8	40 6F 42 11L		
9	F5 5S5 40 5F		
10	L5 1S5 42 12L	from	
11	41 3F L5 ()F	by 8,14,17,24;from 19	
12	32 12L 40 ()F	by 10, 13	
13	F5 12L 42 12L		Square matrix
14	F5 11L 42 11L		
15	F5 3F L0 5F		
16	36 18L L5 3F		

LOCATION	ORDER	NOTES	
17	L4 11L 42 11L		
18	F5 3F 40 3F	from 16	
19	L0 6S5 32 11L		
20	J0 S3 50 20L		
21	26 S9 00 ()F	by 4, 25	Store successive rows of correlation matrix on drum
22	00 ()F L5 6F	by 1	
23	L4 5F 40 6F		
24	42 11L F5 21L		
25	L4 3F 40 21L		
26	F5 5F 40 5F		
27	L4 5S5 F0 6S5		
28	36 10L 22 ()F	by 0	
0	00 830K J0 200F 50 L		
1	26 S9 00 240SS		Store subroutine E on drum
2	00 29F L5 0F		[This is an interlude which is later overwritten.]
3	40 4L 26 999F 26 830N		

Subroutine F

This subroutine reads in rows of the correlation matrix off the drum,
 computes the inverse and stores the rows of the inverse matrix on the drum.

	00 116K
0	K5 F
	42 16L
1	41 8F
	L5 S5
2	42 15L
	46 14L
3	46 19L
	L4 4S5
4	46 26L
	L4 S5
5	42 8F
	L5 32L
6	L4 8F
	40 18L
7	50 16S5
	L5 16S5
8	74 16S5
	S5 F
9	L4 18L
	40 25L
10	40 8F
	L5 33L
11	40 22L
	F5 33L
12	42 23L
	L4 S5
13	46 23L
	00 1F
14	J0 ()F
	L5 L4L

Plant link

by 2

LOCATION	ORDER		NOTES
15	26 35L 00 ()F	by 2	
16	00 1F 22 ()F	by 0	
17	50 150L 50 17L		Read in successive rows of correlation matrix from drum.
18	26 S9 00 F	by 6,20 by 6,20	
19	00 ()F F5 18L	by 3	
20	L4 16S5 40 18L		
21	22 56L 00 1F		
22	L5 ()F 40 ()F	by 11,29 by 11,27	Store successive rows of inverted correlation matrix on drum.
23	L5 ()F 40 ()F	by 13 by 12,28	
24	J0 150L 50 24L		
25	26 S9 00 F	by 9,30 by 9,30	
26	00 ()F F5 23L	by 4	
27	42 22L F5 22L		
28	42 23L L4 4S5		
29	46 22L F5 25L		
30	F4 16S5 40 25L		
31	26 141L 00 F		

LOCATION	ORDER	NOTES	PAGE 21	K 14
32	26 S9			
	00 S7			
33	L5 150L			
	40 S8			
34	00 F			
	00 35L			
35	50 34L			
	26 999F			
36	00 F			
	26 35L			
	26 1N			
	Routine M-14 (modified)			
	00 840K			
0	J0 116F			
	50 L			
1	26 S9			
	00 270SS			
2	00 150F			
	L5 0F			
3	40 4L			
	26 999F			
	26 840N			
	Subroutine G.			
	This program computes and prints out the standard error of regression, the multiple correlation, the standardized and unstandardized regression weights and their standard errors.			
	00 200K			
0	K5 ()F	by 3		
	42 154L			
1	50 10S5			
	75 13S5			
				Plant link

[Note: M-14 has been modified so that the columns of the inverted matrix are scaled by powers of 2 rather than by powers of 10.]

Store subroutine F on drum

[This is an interlude which is later overwritten.]

LOCATION	ORDER		NOTES
2	K5 ()F 00 20F		Compute print parameters for standardized regression weights and standard errors.
3	46 0L 46 123L		
4	L4 4S5 46 2L		
5	50 11S5 75 13S5		Compute print parameters for unstandardized regression weights and standard errors.
6	K5 ()F 00 20F	by 7	
7	46 6L 46 119L		
8	L4 4S5 46 12L		
9	00 1F L5 15S5		
10	L0 16S5 40 0F		
11	19 38F 66 0F		
12	S5 ()F 40 14F	by 8	1/s-n
13	L5 S5 L4 S5		
14	46 19L L0 S5		
15	L4 4S5 46 31L		
16	L5 8F 40 30L		
17	50 SN 50 17L		Read means and standard deviations from drum.
18	26 S9 00 S7		

LOCATION	ORDER		NOTES	PAGE 23	K 14
19	00 ()F	by 14			
	L5 17S5				
20	42 23L				
	F5 17S5				
21	42 25L				
	L5 56L				
22	42 24L				
	L5 14S5	from 155	Dependent variable?		
23	36 148L				
	L5 (t_{ii})F	by 20,148	Diagonal element of inverse		
24	40 9F				
	L5 (s_i)F	by 22,150	Standard deviation		
25	40 10F				
	L5 (c_i)F	by 21,149	Scaling factor		
26	40 11F				
	L5 156L		1/10		
27	40 7F				
	L5 5S5		-1		
28	40 8F				
	32 29L		Waste		
29	50 50SN		Read first row of inverse matrix		
	50 29L		from drum		
30	26 S9				
	00 S7	by 16,152			
31	00 ()F	by 15			
	L5 0L		Set print parameters		
32	46 136L				
	L5 2L				
33	46 134L				
	L5 6L				
34	46 130L				
	L5 12L				
35	46 128L				
	46 146L				
36	41 15F	from 162,166			
	L5 56L				

LOCATION	ORDER		NOTES
37	42 49L L4 S5		
38	42 85L L5 168L		
39	42 48L L4 155L		
40	42 84L 42 127L		
41	L4 155L 42 95L		
42	42 129L L4 155L		
43	42 57L 42 133L		
44	L4 155L 42 77L		
45	42 135L L5 17S5		
46	42 50L F5 17S5		
47	42 51L 41 17F		
48	41 16F L5 (t _{ij})F	by 39,96;from 106	Waste
49	40 3F L5 (s _j)F	by 37,97	
50	40 4F L5 (t _{jj})F	by 46,98	
51	40 5F L5 (c _j)F	by 47,99	
52	40 6F 50 7F		
53	71 3F 40 0F		
54	L3 9F L6 0F		

LOCATION	ORDER		NOTES	PAGE 25
55	36 158L		Change scale	
	L5 OF		Compute standardized regression weight	
56	66 9F		and store	
	S5 SN			
57	40 12F			
	40 (b* _{ij})F			
58	L5 4F			
	L0 10F			
59	40 OF			
	L3 OF			
60	32 77L			
	50 3F			
61	79 3F			
	40 OF			
62	50 OF			
	7J 6F		Compute standard error of standardized	
63	40 OF		regression weights	
	50 5F			
64	7J 9F			
	40 1F			
65	50 1F			
	7J 11F			
66	L4 OF			
	40 OF			
67	50 OF			
	7J 14F			
68	40 OF			
	50 9F			
69	7J 9F			
	40 1F			
70	50 1F			
	7J 6F			
71	40 1F			
	50 OF			
72	71 8F			
	40 2F			

LOCATION	ORDER		NOTES	PAGE 26
73	L7 2F			
	L2 1F			
74	32 163L			
	L5 2F			
75	66 1F			
	41 OF			
76	S5 F			
	50 76L			
77	26 S6			
78	40 (S_{ij}^*)F	by 44,101		
79	40 13F			
80	L3 11S5			
81	36 96L			
82	50 12F			
83	75 10F			
84	40 OF			
85	L7 OF			
86	L2 4F			
87	36 158L			
88	L5 OF		Compute unstandardized regression weight	
89	66 4F			
90	S5 F			
91	32 84L			
92	40 (b_{ij})F	by 40,102		
93	L5 16F			
94	74 (x_i)F	by 38,103		
95	40 OF			
96	LL OF		Compute constant term	
97	36 88L			
98	26 158L		Change scale	
99	L5 OF			
100	L4 15F			
101	40 15F			
102	S5 F			

LOCATION	ORDER		NOTES	PAGE 27
90	40 16F			
	50 13F			
91	75 10F			
	40 0F			
92	L7 0F			
	L2 4F			
93	32 163L		Change scale	
	L5 0F			
94	66 4F		Standard error of unstandardized re-	
	S5 F		gression weight	
95	32 95L		Waste	
	40 ($s_{b_{ij}}$)	by 41,104		
96	F5 48L			
	42 48L			
97	F5 49L			
	42 49L			
98	F5 51L			
	42 50L			
99	F5 50L			
	42 51L			
100	F5 57L			
	42 57L			
101	F5 77L			
	42 77L			
102	F5 84L			
	42 84L			
103	F5 85L			
	42 85L			
104	F5 95L			
	42 95L			
105	F5 17F			
	40 17F			
106	L0 6S5			
	32 48L			

LOCATION	ORDER		NOTES
107	41 OF		
	L5 9F		
108	L0 11F		
	50 12S5		Multiple correlation
109	66 9F		coefficient
	S5 F		
110	40 3F		
	50 110L		
111	26 S6		
	40 5F		
112	L5 5S5		
	L4 3F		
113	40 1F		Standard error of estimate
	L1 1F		
114	32 114L		Waste
	50 114L		
115	26 S6		
	50 2F		
116	7J 10F		
	40 4F		
117	92 139F		
	L3 11S5		
118	32 121L		
	L5 4F		
119	52 ()F	by 7	Print standard error of
	50 119L		estimate
120	26 SK		
	92 131F		
121	92 515F		
	L3 10S5	from 118	
122	32 124L		
	L5 5F		Print multiple correlation
123	52 ()F	by 3	
	50 123L		
124	26 SK		
	92 131F		

LOCATION	ORDER		NOTES	PAGE 29
125	41 3F 92 131F	from 143		
126	92 515F L3 11S5			
127	32 132L L5 ()F	by 40, 138	Print b_{ij}	
128	52 ()F 50 128L	by 35, 161		
129	26 SK L5 ()F	by 42, 139	Print S_{bij}	
130	52 ()F 50 130L	by 34, 165		
131	26 SK 92 965F			
132	32 132L L3 10S5	by 127		
133	32 137L L5 ()F	by 43, 140	Print b_{ij}^*	
134	52 ()F 50 134L	by 33		
135	26 SK L5 ()F	by 44, 141	Print S_{bij}^*	
136	52 ()F 50 136L	by 32, 165		
137	26 SK F5 127L			
138	42 127L F5 129L			
139	42 129L F5 133L			
140	42 133L F5 135L			
141	42 135L F5 3F			
142	40 3F L0 6S5			

LOCATION	ORDER		NOTES	PAGE 30
143	32 125L L3 11S5			
144	32 147L 92 131F			
145	92 515F L1 15F			
146	52 ()F 50 146L	by 35,160	Print constant term	
147	26 SK 92 135F	from 144		
148	F5 25L 42 23L	from 23		
149	F5 23L 42 25L			
150	F5 24L 42 24L			
151	F5 30L F4 16S5			
152	40 30L L5 14S5			
153	L4 14S5 40 14S5			
154	L3 14S5 32 ()F	by 0		
155	22 22L 00 25F			
156	00 F 00 1000 0000 0000J			
157	00 F 00 0100 0000 0000J			
158	50 7F 7J 156L	from 55,82,87		
159	40 7F L5 146L			

LOCATION	ORDER		NOTES	PAGE 31	K 14
160	L4 4S5 42 146L				
161	46 128L L5 134L				
162	L4 4S5 46 134L				
163	26 36L L5 130L	from 73,93			
164	L4 4S5 46 130L				
165	L5 136L L4 4S5				
166	46 136L 50 8F				
167	7J 157L 40 8F				
168	26 36L 00 50SN 00 850K				
0	J0 200F 50 L				
1	26 S9 00 421SS		Store subroutine G on drum [This is a interlude which is later overwritten.]		
2	00 169F L5 0F				
3	40 4L 26 999F 26 850N				
Master Routine					
	00 18K				
0	50 200F 50 L		Read subroutine A		
1	26 S9 00 SS		from drum		

LOCATION	ORDER	NOTES	PAGE 32
2	00 44F		
	50 2L		
3	26 200F	Enter subroutine A	
	22 4L		
4	50 200F		
	50 4L	Read subroutine B and C and H	
5	26 S9	from drum	
	00 700SS		
6	00 191F		
	22 7L		
7	50 S4	Read N-12 from drum	
	50 7L		
8	26 S9		
	00 900SS		
9	00 39F		
	22 10L		
10	50 S6	Read R-1 from drum	
	50 10L		
11	26 S9		
	00 140SS		
12	00 9F		
	50 12L		
13	26 200F	Enter subroutine B	
	50 13L		
14	26 259F	Enter subroutine C	
	22 15L		
15	50 SK	Read P-1 from drum	
	50 15L		
16	26 S9		
	00 150SS		
17	00 28F		
	22 18L		
18	50 200F	Read subroutine D	
	50 18L	from drum	

LOCATION	ORDER	NOTES	PAGE 33
19	26 S9 00 179SS		
20	00 60F 50 20L	Enter subroutine D	
21	26 200F 22 22L		
22	50 200F 50 22L	Read subroutine E from drum	
23	26 S9 00 240SS		
24	00 29F 50 24L	Enter subroutine E	
25	26 200F 22 26L		
26	50 116F 50 26L	Read subroutine F from drum	
27	26 S9 00 270SS		
28	00 150F 50 28L		
29	26 116F 22 30L	Enter subroutine F	
30	50 S6 50 30L	Read R-1 from drum	
31	26 S9 00 140SS		
32	00 9F 22 33L		
33	50 SK 50 33L	Read P-1 from drum	
34	26 S9 00 150SS		
35	00 28F 22 36L		

LOCATION	ORDER	NOTES	PAGE 34	K 14
36	50 200F 50 36L	Read subroutine G from drum		
37	26 S9 00 421SS	Enter subroutine G		
38	00 169F 50 38L			
39	26 200F 24 L	Stop; Begin program again.		
List of Constants				
	00 58K			
0	00 (n)F 00 (n)F			
1	00 S3 00 S3	location of observation row		
2	00 F 00 F	location of means		
3	00 F 00 F	location of sum of cross products		
4	00 1F 00 1F			
5	80 F 00 F			
6	80 F 00 (n)F			
7	00 F 00 S7			
8	00 F 00 ()F	Print parameter for correlations		
9	00 F 00 ()F	Print parameter for covariances		
10	00 F 00 ()F	Print parameter for standardized regression weights		
11	00 F 00 F	Print parameter for unstandardized regression weights		

LOCATION	ORDER	NOTES	PAGE 35
12	00 F 00 1F		
13	00 F 00 10F		
14	00 F 00 F		Dependent variables
15	00 F 00 (s)F		
16	00 F 00 (n)F		
17	00 S8 00 S8 00 116K		
Library Routine R-1	00 125K	Square Root	
Library Routine P-1 (modified)	00 960K		Print One Number Fractional or Integer in a Manner Determined by a Program Parameter
Library Routine N-12	00 860K	Input a Sequence of Decimal Fractions	
0	J0 S6 50 L		Store R-1 on drum
1	26 S9 00 140SS		
2	00 9F L5 0F		[This is an interlude which is later overwritten.]
3	40 12L 00 1F		
4	J0 SK 50 4L		Store P-1 on Drum
5	26 S9 00 150SS		

LOCATION	ORDER		NOTES
6	00 28F		
	L5 OF		
7	40 13L		
	00 1F		
8	J0 S4		
	50 8L		Store N-12 on Drum
9	26 S9		
	00 900SS		
10	00 0039F		
	L5 OF		
11	40 14L		
	26 999F		
	26 860N		
	00 880K		
	Library Routine X-7	Sum Check	Change FF order in X-7 here to read FF 17F
	24 18N		