UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE H 3 - 80

TITLE	Minimization of a Function of Two Variables (DOI or SADOI)
TYPE	Closed
NUMBER OF WORDS	46
TEMPORARY STORAGE	3 words at S3 3 words at S4 3 words at S5
ACCURACY	Depends upon the condition of the function.
DURATION	A "minor" cycle takes T = (1+t) ms where n is the number of "minor" cycles in the "major cycle". Add 1 ms for each decrease in mesh size.

PRESET PARAMETERS

3	00 F 00 aF	where a, a+1, a+2 are the addresses of f(xy), x, y on entry and exit,
4	00 F 00 bF	b, b+1, b+2 are the addresses of δ, δ_x, δ_y,
5	00 F 00 cF	c, c+1, c+2 are working spaces,
6	00 F 00 αJ	α is the factor by which the mesh size is decreased,
7	00 F 00 ϵJ	ϵ is an end constant such that only mesh sizes <u>larger</u> than ϵ will be used,
8	00 F 00 rF	r is the address to the left hand side of which control is transferred after the best values using a given mesh size have been obtained.
9	00 F 00 sF	s is the address of a closed subroutine which takes x from c+1 (1S5), y from c+2 (2S5) and places f(xy) in R_1.

ENTRY

Enter with initial values of x, y, δ in 1S3, 2S3, S4, respectively, and

p	50 pF
p+1	26 qF

q = address of this program

Routine will be left with best values of f(xy), x, y in S3, 1S3, 2S3, respectively.

DESCRIPTION

Define

$f_x \equiv f(x - \delta, y) - f(x + \delta, y)$

$f_y \equiv f(x, y - \delta) - f(x, y + \delta)$

1 - The routine forms f_x and f_y.

2 - $$\delta_x = \frac{1/2\ f_x\ \delta}{1/2\ |f_x| + 1/2\ |f_y| + 2^{-39}},$$

$$\delta_y = \frac{1/2\ f_y\ \delta}{1/2\ |f_x| + 1/2\ |f_y| + 2^{-39}}$$

are formed. Note: The initial mesh size should be chosen so that f_x and f_y do not exceed capacity.

3 - $f(x + \delta_x, y + \delta_y)$ is formed, and tested against the previous $f(x,y)$. If the function has decreased $f(x,y)$ is replaced by $f(x + \delta_x, y + \delta_y)$. The process is then repeated with the same δ_x and δ_y. This constitutes a "minor cycle".

4 - When the above process ceases to improve the function, we return to steps 1 and 2 and compute a new δ_x and δ_y. Steps 1-4 constitute a "major cycle".

5 - When the <u>first</u> "minor" cycle of a given "major" cycle fails to improve the function, indicating that no further improvement is to be expected with the current mesh size, control is transferred to the left hand side of the address in S8. The programmer may here place any program desired: e.g. printing of intermediate results (best current $f(x,y)$, x, and y are always in S3, 1S3, 2S3); alteration of the program depending upon possible knowledge of the function; etc. The program inserted in S8 should terminate by transferring control to the left hand side of (q + 41). If

the programmer does not wish to leave this code, 00 F 00 (q+41)F should be entered in S8 during read-in.

6 - Upon returning to the code, δ, the mesh size will be replaced by $\alpha\,\delta$ where α is entered in S6 during read-in. The steps 1-5 are then repeated.

7 - The steps 1-6 will be repeated (N+1) times where N is the largest number such that $\alpha^{N}\delta_0 < \epsilon$. ϵ is to be entered in S7 during read-in.

NOTES

1 - x_0, y_0, δ_0 The initial guesses for s, y, δ are to be stored in 1S3, 2S3, and S4 before entering this subroutine.

2 - A function, poorly conditioned in having a much larger gradient in one direction (say x) than in the other (say y), can deceive this routine. Such deception can sometimes be penetrated by re-entering the code with a larger mesh size after the code has been left, retaining the best x and y (in 1S3 and 2S3) obtained on the previous entry.

3 - By properly programming the subroutine to compute f(xy) to neglect y, the routine may be used to minimize a function of one variable.

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lgr

LOCATION	ORDER		NOTES	PAGE 1
	00 K(H3)			
0	S5 F		Plant link	
	L4 9L			
1	42 43L			
	L5 1S3		x_0y_0 to 1S5, 2S5	
2	40 1S5			
	L5 2S3			
3	40 2S5			
	50 3L			
4	26 S9		$f(x_0y_0)$ to S3	
	40 S3			
5	L5 11L	From 35	Set switch for failure on first try	
	42 35L	and 43		
6	L5 1S3			
	L4 S4			
7	40 1S5			
	L5 2S3			
8	40 2S5			
	50 8L			
9	26 S9			
	10 1F		1/2 $f(x + \delta, y)$ to 1S4	
10	40 1S4			
	L5 1S3			
11	L0 S4			
	50 S8			
12	40 1S5			
	50 12L			
13	26 S9			
	10 1F		1/2 $f(x - \delta, y)$ in R_1	
14	L0 1S4			
	40 1S4		1/2 f_x in 1S4	
15	L5 1S3			
	40 1S5			
16	L5 2S3			
	L4 S4			

LOCATION	ORDER		NOTES
17	40 2S5		
	50 17L		1/2 f to 2S4
18	26 S9		
	10 1F		
19	40 2S4		
	L5 2S3		
20	L0 S4		
	50 5L		
21	40 2S5		
	50 21L		
22	26 S9		
	10 1F		
23	L0 2S4		
	40 2S4		
24	19 38F		
	L6 1S4		
25	L6 2S4		
	40 S5		
26	L5 1S4		
	66 S5		
27	7J S4		
	40 1S4		δ_x in 1S4
28	L5 2S4		
	66 S5		
29	7J S4		
	40 2S4		δ_x in 2S4
30	L5 1S3	From 40	
	L4 1S4		
31	40 1S5		$y + \delta_y$ in 2S5
	L5 2S3		
32	L4 2S4		
	40 2S5		$y + \delta_y$ in 2S5
33	50 33L		Waste
	50 33L		

LOCATION	ORDER		NOTES PAGE 3
34	26 S9		$f(x + \delta_x, y + \delta_y)$ in R_1 and S5
	40 S5		
35	L0 S3		$f - f$ (before) in R_1
	36 ( )F	By 5 and 39	
36	L5 S5		If negative, success
	40 S3		Replace f_0, x_0, y_0 and set
37	L5 1S5		
	40 1S3		switch for failure after first try
38	L5 2S5		
	40 2S3		
39	L5 20L		and
	42 35L		repeat
40	26 30L		
	00 F		Waste
41	50 44L		
	7J S4		δ_{i+1} in R_1
42	40 S4		and S4
	L0 45L		$\delta_{i+1} - \epsilon$ in R_1
43	36 5L		
	22 ( )F	By 2	Link
44	00 F		$= \frac{\delta_{i+1}}{\delta_i} = \alpha$
	00 S6		
45	00 F		
	00 S7		$= \epsilon$