

UNIVERSITY OF ILLINOIS  
DIGITAL COMPUTER

ILLINOIS CODE H 2 - 72

TITLE A search for the Real Roots of  $f(x) = 0$  (DOI or SADOI)

TYPE Closed

NUMBER OF WORDS 80

TEMPORARY STORAGE 7 to 16 inclusive

PARAMETERS Several Program parameters must be set.

ACCURACY Depends upon the function  $f(x)$  and the error in the routine which computes it. Possible error inherent in this routine is about  $2^{-38}$ .

DURATION  $[2 + (2.33 + 2t)n + (6 + t)ir]$  ms where  $t$  is the duration in ms of the routine which computes  $f(x)$ .  $n$  is the number of intervals examined for roots,  $i$  is the average number of iterative linear interpolations necessary to find a root, and  $r$  is the number of roots found.

READ AROUND Depends upon routine used to compute  $f(x)$ , whose duration is denoted by  $t(\text{ms})$ . RAR adjacent to routine is  $22.5/(2.33 + 2t)$  or  $22.5/(6+t)$  whichever is greater. Next to the temporary store it may be 8 times as great.

DESCRIPTION This routine is a combination of Code HL (Inverse Interpolation), Code P2 (Decimal Fraction Print), plus orders which examine an arbitrary range of the independent variable for roots of  $f(x) = 0$ . In  $n$  steps of length  $h$ , the routine examines the interval  $x_0 < x \leq x_0 + nh$  for roots. The  $i$ th interval is regarded as  $x_0 + (i-1)h < x \leq x_0 + ih$ . If such an interval contains an even number of roots. (e.g., 0) none will be found, if it contains an odd number of roots (e.g., 1) one will be found. This means the programmer must have sufficient information about  $f(x)$  to choose  $x_0, n$ , and  $h$  compatible with both economy of time and the finding of all the roots. Multiple

roots are treated as single roots.

The routine is entered with  $x_0$  in  $R_1$  and the orders:

p	J0 kF or 50 rF 50 pF
p+1	26 qF 00 nF
p+2	00 F 00 hJ
p+3	00 aF -- --

where  $q$  is the address of this routine,  $n$  is the number of intervals to be examined,  $h$  is the length of an interval, and  $a$  is the address of the closed subroutine which replaces  $x$  in  $R_1$  by  $f(x)$  in  $R_1$ . If the first parameter is J0 kF, the roots will be printed with sign in a column to  $k$  decimal digits. If it is 50 rF, the roots will be stored sequentially starting at memory position  $r$ . If  $x_i$  be the roots found, then  $\dots, < x_{i-1} < x_i < x_{i+1} < \dots$ , always. Control will be returned to the right side of  $p+3$  after the  $n$  intervals have been examined.

NOTES

If it is never desired to print the roots, the last 18 words of this routine (62 - 79) may be overwritten. Further, if for other purposes it is desired to use Code P2 (Decimal Fraction Print), the one incorporated in this program may be utilized in the usual fashion by merely transferring control to the left hand side of the 62nd word.

RT: 1/23/59
DATE July 20, 1954
CODED BY J. N. Snyder
APPROVED BY J. P. Nash

LOCATION	ORDER		NOTES	PAGE 1	H 2
	00 K(H2)				
0	40 10F		Store $x_0$		
	S5 F		Move in link		
1	L4 61L		Plant n's address		
	42 10L				
2	L4 61L		Plant h's address		
	42 12L				
3	L4 61L		Plant call for auxiliary routine		
	42 8L		address		
4	42 16L		Plant link address		
	36 7L				
5	46 59L				
	L5 57L		Set store or print orders		
6	42 53L				
	22 8L				
7	46 55L	From 4			
	L5 54L				
8	42 53L				
	L5 (p+3)F	By 3	From 6		
9	46 18L				
	46 21L				
10	46 32L				
	L5 (p+1)F	By 1			
11	10 20F		Plant step counter		
	01 20F				
12	40 8F				
	L5 (p+2)F	By 2	h to 9		
13	40 9F				
	L4 10F		Set first interval end points		
14	40 12F				
	L5 8F	From 53, 57, 60			
15	L0 61L		Test for end		
	40 8F				
16	36 17L				

LOCATION	ORDER		NOTES	PAGE 2	H 2
17	22 (p+3)F L5 10F 50 17L	By 4 From 16	Link		
18	26 ( )F 10 1F	By 9			
19	40 11F L5 12F				
20	40 7F 50 20L				
21	26 ( )F 10 1F	By 9			
22	40 13F L3 13F				
23	36 49L 50 13F				
24	75 11F 36 48L				
25	L5 11F L0 13F	From 45			
26	40 14F 50 10F				
27	75 13F 40 16F		Search for a root		
28	S1 F 50 12F				
29	74 11F L0 16F				
30	66 14F S5 F				
31	40 14F 50 31L				
32	26 ( )F 10 1F	By 10			
33	40 15F				

LOCATION	ORDER		NOTES	PAGE 3	H 2
34	L3 15F 36 47L L5 10F				
35	L0 12F 40 16F				
36	51 22L 00 2F				
37	L2 16F 36 47L				
38	50 15F 75 11F				
39	32 41L L5 11F				
40	10 1F 40 11F				
41	22 43L L5 12F	From 39			
42	40 10F L5 13F				
43	40 11F L5 14F	From 41			
44	40 12F L5 15F				
45	40 13F 26 25L				
46	80 F 00 F		= -1		
47	L5 14F 22 49L	From 34,37			
48	L5 46L 22 49L	From 24			
49	L5 12F 40 16F	From 23	Result in 16		

LOCATION	ORDER		NOTES	PAGE 4	H 2
50	L5 7F 40 10F		Advance test interval		
51	L4 9F 40 12F				
52	L7 16F 32 53L		Test result, if < 0 repeat		
53	22 14L 26 ( )F	By 6, 8	From 50		
54	L5 16F 50 54L	From 53	Store root and advance address of store order		
55	40 ( )F L5 55L	By 7			
56	L4 28L 46 55L				
57	22 14L 00 58L				
58	92 129F L5 16F	From 53	Line feed and print root		
59	50 ( )F 50 59L	By 5			
60	26 62L 22 14L				
61	00 F 00 1F		Unit of count		
62-79			Code P2, Decimal Fraction Print		