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Cwynar et al.

[54] **SINE-COSINE FUNCTION GENERATOR USING A POWER SERIES**

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[52] **U.S. Cl.** **235/197,** 235/186, 235/194

[51] **Int. Cl.** **G06g 7/22,** G06g 7/26

[58] **Field of Search**.........235/197, 150.53, 193, 194, 235/183, 182, 180, 186, 189

[56] **References Cited**

UNITED STATES PATENTS

3,018,052	1/1962	Fogarty	235/197
3,022,009	2/1962	Fogarty	235/193
3,205,349	9/1965	Bryan et al.	235/197
3,457,394	7/1965	Grado	235/197 X

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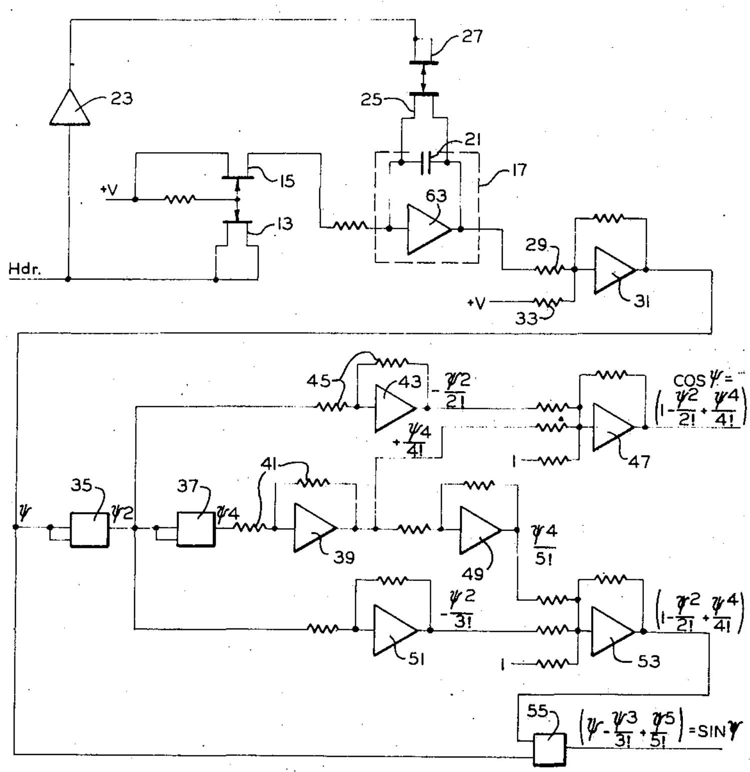
[57] **ABSTRACT**

A method and apparatus for generating sines and cosines of an angle at high frequencies using power series. By using the terms computed for cosine series in computing the sine series a significant reduction in the number of multipliers required is achieved.

4 Claims, 2 Drawing Figures



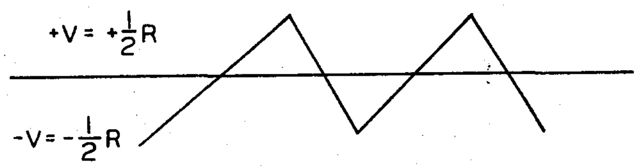

+V = +1/2R
-V = -1/2R

FIG. 1

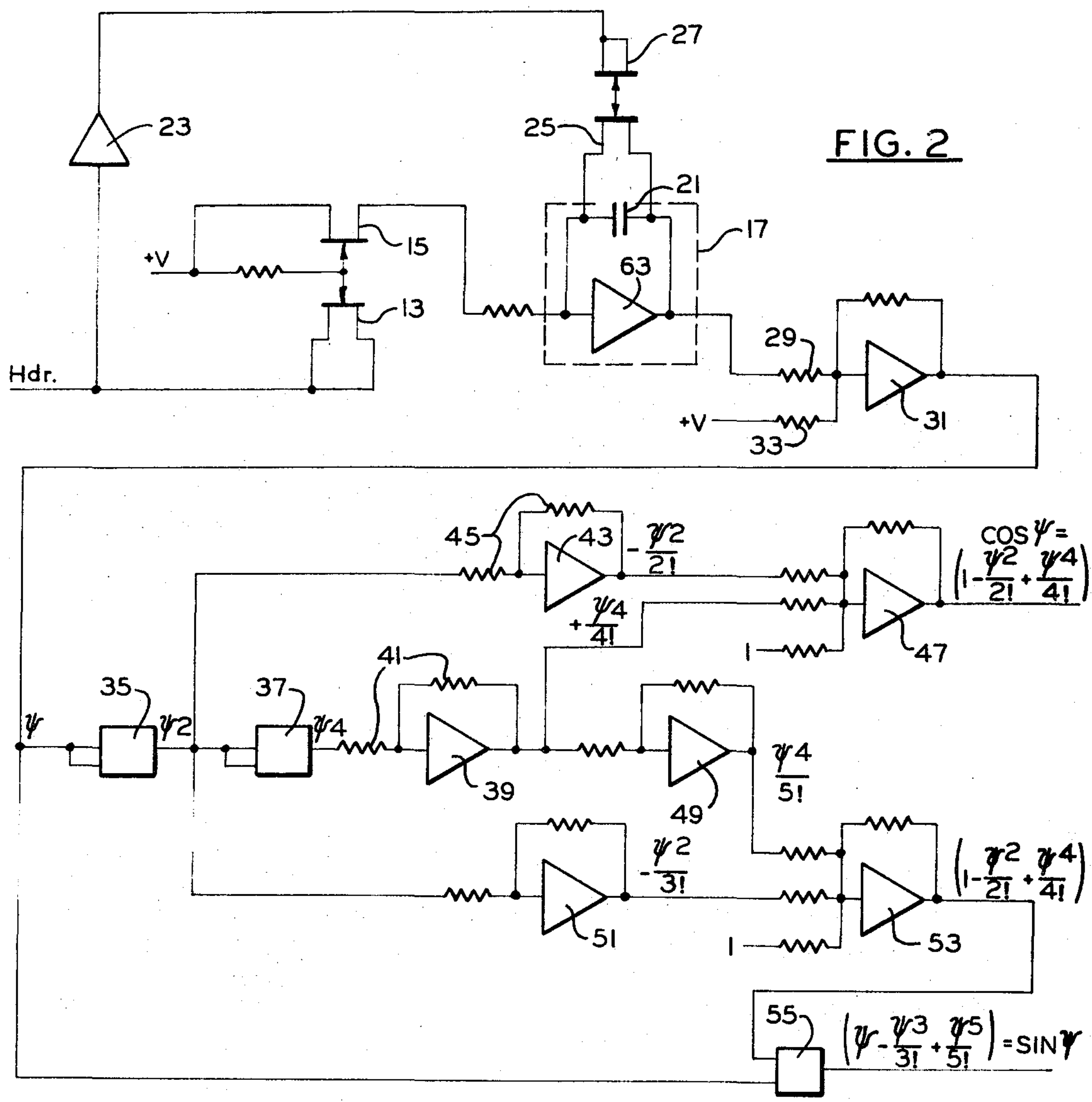

27
23
25
21
15
17
+V
63
13
29
Hdr.
31
+V
33
45
43
$-\frac{\psi^2}{2!}$
$\cos\psi = \left(1-\frac{\psi^2}{2!}+\frac{\psi^4}{4!}\right)$
$+\frac{\psi^4}{4!}$
1
47
41
35
37
ψ
ψ^2
ψ^4
39
49
$\frac{\psi^4}{5!}$
$-\frac{\psi^2}{3!}$
$\left(1-\frac{\psi^2}{2!}+\frac{\psi^4}{4!}\right)$
51
1
53
55
$\left(\psi-\frac{\psi^3}{3!}+\frac{\psi^5}{5!}\right) = \sin\psi$

FIG. 2

SINE-COSINE FUNCTION GENERATOR USING A POWER SERIES

This invention relates to function generation and more particularly to an improved method and means for generating the sine and cosine of an angle.

In many analog computing situations it is necessary to generate the sines and cosines of angles. One such situation is disclosed in U.S. Pat. No. application Ser. No. 108,447 filed by Irzeciak, Millard and Woycechowsky on even date herewith and assigned to the same assignee as the present invention.

In the past most sine-cosine computations have been done using servos driving sine-cosine potentiometers. Such a method works well within the limits of the servo. However, where fast response, beyond that of servos is required a different method must be used. The use of sines and cosines in the above referenced applications is such a case since computation at television scan rates is required.

Another method of generating sines and cosines is with a power series. This is shown in *Analogue Computation* by Stanley Fifer Vol II p 418–20 (McGraw Hill Book Company, Inc. 1961). As shown therein the required multiplications are done using servos. However, by replacing the servos with high speed multipliers the response can be made to meet high speed requirements. Since presently available multipliers are expensive, reduction of the number required in a given system can greatly reduce its cost. The present application shows how a sine-cosine function generator capable of high speed operation may be constructed using a minimum number of multipliers.

It is the object of this invention to provide a sine-cosine generator capable of high speed operation.

Another object is to provide a sine-cosine function generator using fewer multipliers than is possible with prior generators of this type.

A further object is to provide a sine-cosine generator which is useful in generating certain types of visual display.

Other objects of the invention will in part be obvious and will in part appear hereinafter.

The invention accordingly comprises the several steps and the relations of one or more of such steps with respect to each of the others, and the apparatus embodying features of construction, combinations of elements and arrangement of parts which are adapted to effect such steps, all as exemplified in the following detailed disclosure, and the scope of the invention will be indicated in the claims.

For a fuller understanding of the nature and objects of the invention reference should be had to the following detailed description taken in connection with the accompanying drawings, in which:

FIG. 1 shows a saw tooth wave scaled to represent an angle; and

FIG. 2 is a schematic diagram of the preferred embodiment circuitry for practicing the present invention.

A practical use of a sine-cosine function generator is shown in the above referenced U.S. patent application. In the system shown therein the angles involved relate to the azimuth and elevation angles by which a spot on a *tv* raster is displaced from the center of the raster. Thus, if the horizontal field of view displayed on display is, for example, one radian, then the horizontal angular travel of a spot on the raster will be from minus one half radian to plus one half radian. Therefore, a sawtooth wave which is synchronized with the horizontal scan of the raster can be scaled to represent plus one one half radian to minus one half radian.

FIG. 1 show such a waveform. The wave starts at a voltage of minus V scaled to equal minus one half R and linearly increases to a plus voltage representing plus one half R at which time it returns to minus V. This represents the retrace of the horizontal scan on the raster.

The circuits used to obtain this waveform are shown in the upper portion of FIG. 2. A horizontal or vertical drive signal (H dr.) is an input to FET 13 which has its source and drain tied together and acts like a diode. It is used in this case (other than a diode) to match the impedance of FET 15.

The H drive signal shown will be on during the horizontal sweep time and off during the retrace. Thus the input to FET 15 through FET 13 will turn on FET 15 and provide a voltage input to integrator 17 comprising amplifier 63 and capacitor 21. The input voltage will be integrated and the output of integrator 17 will be a gradually increasing voltage. When the H drive goes off during retrace, the signal will be inverted through inverting amplifier 23 and will turn on FET 25 through FET 27. This will provide a path to discharge capacitor 21. Thus the capacitor will be alternately charged and discharged providing a saw tooth output from integrator 17 which is synchronized with the horizontal raster scan. Proper selection of components using techniques well known in the art will result in reaching the desired scaled voltage before the discharge of capacitor 21.

The output of integrator 17 is provided through resistor 29 as an input to summing amplifier 31. The second input through resistor 33 is a positive voltage which when inverted through the amplifier will cause a negative offset of the output of integrator 17 and cause the sawtooth wave to swing plus and minus as shown on FIG. 1.

The remainder of the circuits shown on FIG. 2 are used in computing the sine and cosine of the angle represented by the output of amplifier 31 i.e. the wave of FIG. 1, using the equations described below:

The sine and cosine of an angle may be expressed by the following power series:

$$\text{Sine } \Psi = \frac{\Psi^3}{3!} + \frac{\Psi^5}{5!}$$

$$\text{Cos } \Psi = \frac{\Psi^2}{2!}\ \frac{\Psi^4}{4!}$$

The number of terms in each of the series go on to infinity. However, using the expression with only the terms shown above will be sufficiently accurate for most applications. The most obvious way to implement these equations would be to provide analog multipliers and adders for each of the operations expressed in the equation. However sine Ψ can also be expressed as:

$$\text{Sin } \Psi = \Psi\left(1 - \frac{\Psi^2}{3!} + \frac{\Psi^4}{5!}\right)$$

The similarity of the portion of the equation in the brackets to the cosine expression makes it possible to compute both the sine and cosine using fewer multipliers than would otherwise be necessary.

As shown on FIG. **2**, a value representing the angle Ψ is supplied as both inputs to a multiplier **35** having an output of Ψ^2. (In the actual case the output might be $\Psi^2/10$ but it is obvious that the proper scaling may be maintained using standard analog techniques. Thus, an output of Ψ^2 will be assumed.) The Ψ^2 output of multiplier **35** provides the two inputs to multiplier **37** resulting in an output of Ψ^4. In operational amplifier **39** Ψ^4 is multiplied by 1/4! through the proper scaling of resistors **41**, In like manner another Ψ^2 output from multiplier **35** is multiplied by 1/2! in amplifier **43** with the resistors **45** providing the proper scaling. The input to amplifier **39** is provided to the non-inverting input providing a positive output and that of amplifier **43** to the inverting input to obtain a negative output. The $\Psi^2/2$ and $\Psi^4/4!$ outputs are summed in amplifier **47** along with a value of 1 to obtain the desired output of

$$\cos \Psi = \left(1 - \frac{\Psi^2}{2!} + \frac{\Psi^4}{4!}\right)$$

Similarly the quantity

$$\left(1 - \frac{\Psi^2}{3!} + \frac{\Psi^4}{5!}\right)$$

is obtained using amplifiers **49** to multiply $\Psi^4/4!$ by one fifth and **51** to multiply Ψ^2 by 1/3!, and then summing the results with 1 in amplifier **53**. This result is then multiplied by Ψ in multiplier **55** to obtain sin Ψ equal to

$$\left(\Psi - \frac{\Psi^3}{3!} + \frac{\Psi^5}{5!}\right)$$

In this way the previously computed values of Ψ^2 and Ψ^4 are used to obtain the final result with a reduction in the number of multipliers required.

Thus, a method and apparatus for determining with reasonable accuracy the values of the sine and cosine of an angle at television frequencies with a reduced number of multipliers has been shown. Although a saw tooth generator which has very good accuracy and linearity because of the use of FETS has been shown, any standard generator such as a TV sync generator may be used. The method may also be extended for use with power series expressions of more than three terms in which case corresponding reductions in the number of multipliers will result. For example, with four terms only four multipliers are needed. In comparison, when done separately as shown in the above textbook reference three multiplications for the sine and two multiplications for the cosine would be required for expressions of three terms. With four terms a total of seven multiplications, as compared with four under the present method, would result. As noted above, multipliers are expensive and reduction of two or three in the number of required multipliers can greatly reduce the system cost.

What is claimed is:

1. Apparatus to generate the sine and cosine of an angle at high frequencies comprising:

a. signal generating means to generate a first signal representing the angle for which a sine and cosine is to be generated;

b. a first multiplier having as both inputs said first signal and having an output representing the square of said angle;

c. a second multiplier having as both inputs the output of said first multiplier and having an output representing the fourth power of said angle;

d. a first amplifier having the output of said first multiplier as its input and scaled to multiply said input by one over two factorial;

e. a second amplifier having the output of said second multiplier as its input and scaled to multiply said input by one over four factorial;

f. a third amplifier having as a first input the output of said first amplifier, as a second input the output of said second amplifier and as a third input a voltage representing a value of one whereby the output of said third amplifier will be a power series approximation of the cosine of said angle;

g. a fourth amplifier having the output of said first multiplier as its input and scaled to multiply said input by one of three factorial;

h. a fifth amplifier having the output of said second multiplier as its input and scaled to multiply said input by one over five factorial;

i. a sixth amplifier having as a first input the output of said fourth amplifier, as a second input the output of said fifth amplifier, and as a third input a voltage representing a value of one; and

j. a third multiplier having as a first input the output of said sixth amplifier and as a second input said first signal, whereby the output of said third multiplier will be a power series approximation of the sine of said angle.

2. The invention according to claim **1** wherein said signal generating means is a saw tooth generator.

3. The invention according to claim **2** wherein said generator is a television horizontal sweep generator.

4. The invention according to claim **2** wherein said generator is a television vertical sweep generator.

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