Digital Computer Laboratory
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

SUBJECT: BIWEEKLY REPORT, DECEMBER 13, 1953

To: Jay W. Forrester

From: Scientific and Engineering Computation Group

1. MATHEMATICS, CODING AND APPLICATIONS

1.1 Introduction

During the period covered by this report 238 coded programs were run on the time allocated to the Scientific and Engineering Computation (S&EC) Group. These programs represent part of the work that has been carried on in 28 of the problems that have been accepted by the S&EC Group. Progress on 23 of these problems is given below in terms of programming hours, minutes of computer time, and progress reports as submitted by the programmers in question.

Two new problems were initiated during this period. Problem #153 seeks to determine the gust response of a flexible swept-wing airplane under various conditions. This work is being carried out by K. Foss of the Aeronautical Engineering Department with the assistance of D. Sternlight of the Mathematics Department. In problem #157, Dr. A. Meckler of the Solid State Physics Group has developed routines for multiplying rectangular matrices of arbitrary order (up to 32).

A revised version of the programmed arithmetic (PA) section of the comprehensive system (CS) has been developed to make available a significant amount of mistake anticipation, an unlimited number of cycle counters, and an extended and improved use of buffer registers. The new version of the PA will not be put into general use for at least one month but will be made available to programmers upon special request and under staff supervision. This time delay will provide for more extensive testing of the routines and for a more gradual transition from the old PA to the new.

Dr. J.H.Laning Jr. of the Instrumentation Laboratory spoke at the Seminar on Computing Machine Methods on December 1, 1953. He described a program (that he and N. Zierler have developed) capable of accepting mathematical equations expressed on Flexowriter punched paper tape in ordinary mathematical notation (within certain limits imposed by the Flexowriter)as input and automatically providing the desired solutions.

The CS introductory programming course was begun on December 7, 1953 as announced. A report on the course is given below under problem #100.

1.2 Programs and Computer Operation

The following summary is included as a guide for interpreting the abbreviations used below. A more detailed description of the terms involved can be found in M-2497.

a. The upper case letter following the problem number has the following significance:
A implies the problem is NOT for academic credit, is UNsponsored.
B implies the problem IS for academic credit, is UNsponsored.
C implies the problem is NOT for academic credit, IS sponsored.
D implies the problem IS for academic credit, IS sponsored.

The absence of a letter indicates that it is an internal S&EC problem.

b. DIC denotes the Division of Industrial Cooperation.
DCL denotes the Digital Computer Laboratory.
CMMC denotes the Committee on Machine Methods of Computation.
DDL denotes the Division of Defense Laboratories.

100. Comprehensive System of Service Routines, developed by the S&EC Group at the Digital Computer Laboratory for the input conversion of suitably prepared punched paper tapes. When so requested, these routines automatically provide a program with suitable programmed arithmetic, cycle-counting, and output facilities.
:DCL Staff: Arden, 12 hours; Best, 31.75 hours; Combelic, 6 hours; Demurjian, 8.5 hours; Denman, 16 hours; Frankovich, 26 hours; Hazel, 16 hours; Helwig, 80 hours; Kopley, 3 hours; Porter, 21 hours; WWI, 463 minutes

Testing of the new CS continues. Both the new programs and the old program are presently recorded on magnetic tape unit 0. Conversion of a program by means of the new CS can be obtained only if it is specifically requested.

A memorandum (M-2553) has been written describing the differences between the new (30-j,j) PA and the old. Programmers who plan to use the new PA should read this memorandum before using the program.

Work has been started on a PA post-mortem for the revised (30-j,j) PA.

A description of the programs in the CS which involve PA is being written and will be included in CS manual 3.

Helwig

All of the modifications previously described for the "new" CS are now working and are included in the "new" CS. In addition, a modification has been successfully tested which will enable CS to convert OCTAL programs. The details are described in a forthcoming Bulletin Board Memo. One minor change in the unassigned flad table remains to be made, however, before work can be begun on the "Direct" CS.

Frankovich

The Flad Table routine was tested along with J. Frankovich's test for unassigned flads. The floating address assignments may be printed in either the octal or decimal number system.

Corrections and improvements were made on the drum post-mortem subroutines. Three tests were made using different entry points to the subroutines.

Hazel

The new PA worked for (24,6) numbers in each of two test routines. These test routines will be modified to test other number systems, especially (15,15) and (30,0).

Programs are being written to generate every type of PA alarm in order to test the PA-PM.

Best

The CS introductory programming course was resumed on December 7 with an enrollment of 17 students. Of these students, seven are working on problems that have already been accepted for solution on WWI, two expect to submit problems in the very near future, two are from Group 61, and the remaining six are associated with the S&EC Group.

The course consists of 20 one-hour lectures given by members of the S&EC Group over a period of two weeks. The material for the first half of the course has been described in some detail in the eight completed chapters of CS Manual I which is serving as a text for the course. It is expected that the remaining ten lectures will soon be written up to complete CS Manual I.

It is planned to repeat the course during the first two weeks of each month provided there is a sufficient number of suitably qualified applicants.

Denman, Kopley, Porter

101 C. Optical Properties of Thin Metal Films on transparent backings are determined and printed out automatically by this program; the input data consist of the observed reflection and transmission coefficients, the index of the backing, the wavelength, and the sample thickness. The program calculates by means of an iterative procedure and prints out the index of refraction and the absorption coefficient of the film, the rate of variation of these constants with reflection and transmission, and the film's conductivity and dielectric constant.

:for Professor L. Harris, Chemistry Department, Dr. A.L.Loeb
:by Dr. A.L.Loeb, (DIC), 15 hours; J. Richmond (DIC), 20 hours;
:DCL Staff: Denman, 1 hour; WWI, 59 minutes

Experimental results published by Krantkrämer for thicknesses down to 17Å were used as input data, and even the thinnest samples gave satisfactory runs in contrast to the samples reported on by Goos. It was discovered that Goos' experimental setup was such as to attempt to eliminate multiple reflection in the backing. His results for thinnest films were therefore less than the minimum results predicted by the standard equation, so that no set of n and k could be found to satisfy Goos' experimental values for the thinnest films. This is the reason for the apparent lack of conversion of the program for thinnest films. The program now runs satisfactorily for the entire range of values tested so far.

106 C. MIT Seismic Project is concerned with the development of methods for locating deep reflections from underground strata in seismic prospecting. The basic method is one of prediction by means of an optimum linear operator.

:for Professor P.M.Hurley, Geology and Geophysics; Professor G. Wadsworth, Mathematics Department

:by E.A.Robinson(Res. Assoc.), H. Briscoe, 36 hours; S. Simpson, 20 hours; W. Walsh, 25 hours

:DCL: WWI, 46 minutes

In the last two weeks the group has concentrated on programming rather than production. The forward solution of symmetric simultaneous equations has been successfully coded, using Gauss Elimination and (24,6) arithmetic, and is being refined.

Recent emphasis in the group has been on viewing Linear Operators as filtering mechanisms. This study involves new programs using Fourier rather than correlation techniques, and these programs are under way.

108 C. An Interpretive Program is being developed that will accept algebraic equations, differential equations, etc. expressed on Flexowriter punched paper tape in ordinary mathematical notation (within certain limits imposed by the Flexowriter) as input and automatically provide the desired solution.

:for Dr. J.H.Laning,Jr., Instrumentation Laboratory

:by Dr. J.H.Laning, Jr.(DIC), N. Zierler(DIC)

:DCL Staff: Hazel, 2 hours; Siegel; WWI, 12 minutes

Two problems were solved with very satisfactory results using the interpretive routine.

One of these produced the solutions of a number of independent quadratic equations, printing both of the real roots or the conjugate imaginary roots as the nature of the equation required. This problem was solved as a test of the routine.

The second problem involved the evaluation of a polynomial function,

$$P_L(\eta) = P_{L-1}(\eta)\,\frac{(2L+2)^2+2\eta^2}{(2L+3)(2L+2)^2(2L+1)^2}$$

with $P_0(\eta) = 2\eta$ and $\eta = 0.5$ for several values of L. The evaluation of this function is connected with the work being done for Problem 122.

109 C. An Airplane Pursuit-Course Program is being developed which will take account of airplane dynamics and projectile ballistics and thus determine an airplane pursuit course in three dimensions. The problem consists essentially of solving 14 simultaneous non-linear differential equations by the Runge-Kutta Method which is of fourth-order accuracy.

:for Mr. J.B.Feldman, Instrumentation Laboratory

:by M.H.Hellman(DIC), 40 hours

:DCL Staff: Porter, 2.5 hours; WWI, 99 minutes

A horizontal two-dimensional test run requiring 30 minutes and 540 steps was made to compare the accuracy of the two-dimensional horizontal pursuit course with that of the three-dimensional pursuit course. The results obtained indicate the two-dimensional horizontal pursuit course is a good approximation of the three-dimensional pursuit course whose initial conditions are in the horizontal plane.

It is planned to make a number of three-dimensional pursuit course runs with the airplane and target initially in a slant plane. From the results of these runs it will be possible to evaluate the errors in the calibration of existing gunsights.

113 C. A Stress Analysis of an L-shaped Homogeneous Planar Structure is being made for the case of a concentrated static load. This structure is approximated by a framework of bars which will deform in the same manner as the prototype. This framework is then analyzed using the principles of virtual work and Southwell relaxation techniques. Boundary conditions have been specified for the edge of the framework so that the deformations of the model will conform to the actual deformations of the structure.

:for Professor J.S.Archer, Department of Civil and Sanitary Engineering
:by S. Sydney (Res. Assist. CMMC), 60 hours
:DCL Staff: Kopley, 2 hours; WWI, 359 minutes

The program has been tested in sections and is operating satisfactorily. The individual units will be combined into a master tape and tested as a whole before production runs are made.

122 B. Coulomb Wave Functions. Regular and irregular solutions of the radial Schrödinger equation with a coulomb potential are being sought. These solutions must have the proper asymptotic form to correspond to scattered waves from such a potential. The solutions are written as a power series in the radial variable and the coefficients are determined by recursion formulae.

:for Professor H. Feshbach, Professor P.M.Morse ,Physics Department
:by A. Temkin (Res. Assist.CMMC), 1 hour
:DCL Staff: Siegel, .5 hour; WWI, 9 minutes

A program has been constructed for finding $P_L(\eta)$, a quantity which appears in the recursion relation for the power series coefficients a_n^L. The relation for $P_L(\eta)$ is

$$\frac{P_L(\eta)}{2\eta} = \frac{1 \cdot (1+\eta^2)(4+\eta^2) \; . \; . \; . \; . \; . \; (L^2+\eta^2) \; 2^{2L}}{(2L+1)\left[(2L)\,!\right]^2}$$

The program tested the value $\eta = .5$ and went from L=0 to L=6. The latter value was 9.6×10^{-14} and since the values of $\eta \leq 30$, the program shows that this term can be left out of the recursion formula for values of $L \geq 6$. It is my immediate prospect to write a program to find the a_n^L.

131. <u>The Training of New Personnel, Tours and Demonstrations</u> are among those activities included in this problem. Generally speaking, any approved staff problem relating to training and/or demonstrations is considered to be in this category.
:DCL Staff: <u>Kopley</u>, 2 hours; WWI, 37 minutes

Seven students from Professor Crandall's class at MIT joined Professor Crandall in a tour of the WWI installation on December 1 between 5 and 6 P.M. Four other interested outsiders joined the tour which included computer and Flexowriter demonstrations.

132 C. <u>Subroutines for the Numerically Controlled Milling Machine</u> are being revised and tested. The set of subroutines facilitates programming of the computations involved in the preparation of numerical data used to control the milling machine. The subroutines involve routine numerical and logical operations.
:for J.O.McDonough, Servomechanisms Laboratory, DIC Project 6873
:by <u>J.H.Runyon</u> (E.E.Res. Assist.), 10 hours
:DCL: WWI, 25 minutes

A routine for finding points on asymmetrical series 16 airfoil cross-sections was successfully tested. Also successfully tested was a routine for finding offsets normal to plane curves. This process is used in the first-mentioned routine.

Routines for cut-length selection for constant tolerance and for finding the first two derivatives of the series 16 section are still being tested.

134 C. <u>Numerical Diagonalization Procedure.</u> This program computes the eigenvalues and eigenvectors of a symmetric matrix by a method of successive rotations. The program is available for use in any problem in which this calculation is required.
:for Professor J.C.Slater, Physics Department
:by <u>A. Meckler</u>(DIC), 1 hour
:DCL Staff: Arden, 12 hours; WWI, 90 minutes

The secular equation routine is being modified so that the main program will be independent of particular data and can therefore be converted once and for all. The matrices will be on a separate tape and will be fed in automatically for production runs.

Also, 165 5 x 5 matrices are being diagonalized for Professor J.C.Slater and Dr. G.F.Koster, who are doing an electron energy band calculation for body-centered structures.

136. <u>Matrix Equations.</u> Various methods have been studied for the solution of a set of linear algebraic equations. A variation of the Hestenes-Stiefel conjugate gradient method has been programmed and tested for insertion in the S&EC Library of Subroutines.
:DCL Staff: <u>Arden</u>, 14.5 hours; WWI, 81 minutes

E. Craig of the Electrical Engineering Department is conducting tests to determine the effects of bad matrix donditioning on the accuracy of the gradient method that was developed into a subroutine as part of this problem.

An attempt is being made to solve a system of order 30 using this subroutine.

137 D. Investigation of Atmospheric Turbulence as a noise input to airborne control systems. A stationary random process is assumed so that the methods of generalized harmonic analysis may be used to describe the turbulence components in terms of their power spectral densities.
:for Professor R.C.Seamans, Department of Aeronautical Engineering
:by R.A.Summers (Res. Assist.), 6 hours
:DCL: WWI, 141 minutes

About one-half of the required Fourier transform runs have been completed and it is anticipated that about one more hour of computer time will complete problem 137.

140. Summer Session System consists of a conversion program, an interpretive routine, and mistake diagnostic routines stored in WWI. A special mnemonic instruction code has been developed for use with this system thus simulating a computer with characteristics quite different from those of WWI. This Summer Session (SS) computer was developed for the use of students participating in the MIT 1953 summer session course on "Digital Computers and Their Applications". The SS computer is being used in the E.E.Department courses 6.535 and 6.25 and is available to programmers with suitable problems.
:DCL Staff: Best, 33.5 hours; Combelic, 26 hours; Frankovich, 2.5 hours; Hoy, 2.5 hours; Siegel, 47 hours; WWI, 516 minutes

Considerable effort has been expended to eliminate errors in the SS conversion program and in the instruction rin, both of which must convert numbers which appear as a sequence of Flexowriter characters on a punched paper tape into a form suitable for storage in an SS memory location. The reasons for the failure of these programs to treat numbers containing "x10^{a}" have been determined, and the routines will be corrected in the near future. In addition, a correction has been made in the conversion program to permit the use of "a7±3| ", etc., at the end of an SS program to modify the contents of a register which had been filled incorrectly in the body of the program. Heretofore, only registers which had been assigned a floating address or whose absolute address was known could be changed directly by using this device.

Much WWI time was devoted to running SS programs written by students in Course 6.25. The operation of the SS computer was entirely satisfactory. For the convenience of these students, the SS computer was temporarily modified to display automatically a set of calibrated axes and the names of the programmers on the oscilloscope and to photograph these on the same frame which was to contain the curve plotted by each student's program.

The automatic title-display routine has been completed and incorporated into the SS computer. The title of the program is now displayed on all output media selected by instructions appearing in the program.

141. S&EC Subroutine Study has been undertaken for the final testing of subroutines selected for incorporation into the Library of Subroutines. Although very little effort is going into the specific development of subroutines, programs that have been written for other S&EC problems and seem to be of general use will be suitably modified for the S&EC Library.
:DCL Staff: Best, 3 hours; Denman, 21 hours; Siegel, .25 hour; Vanderburgh, 1 hour; WWI, 14 minutes

Since the CS computer will soon be altered to process OCTAL as well as DECIMAL programs, the problem of avoiding a dual Subroutine Library has come up. In order that only one copy of a Library Subroutine need be filed, the following measures were adopted:

All Library Subroutines will start with the characters LSR and will end with the words END OF SUBROUTINE. All Library Subroutines will be written with DECIMAL addresses. The symbol LSR will indicate to the conversion routine that whatever follows is DECIMAL. The symbol "END OF SUBROUTINE" will indicate that the original number base is to be used again.

Personal subroutines submitted with the request "Type in Subroutine form" will be typed with LSR at the beginning and END OF SUBROUTINE at the end. They will be filed under the programmer's own tape number.

Library Subroutines will be filed in a separate file under their LSR number. Routines under test for inclusion in the LSR will be filed under problem 141 in the Main File. It is proposed that LSR routines be filed in eight major categories according to the code shown below: Integer numbers will be used within the categories. The objective here is to keep the system as simple as possible and yet allow some indexing other than alphabetical.

OT.......Typewriter (Direct Printer)
OD.......Delayed Typewriter (Magnetic Tape)
OTD......Both Delayed and Direct Typewriter
OS....... Scope (and Camera)
FU.......Function Evaluation
MA.......Matrix (Inversion, Eiganvalues, etc.)
DE.......Differential Equations
SP.......Special (For any routine that does not fit in the other categories)

List of Available Subroutines for OCTAL or DECIMAL use.

			Tape Numbers
LSR	T1	(24,6)Print	3313
LSR	TD1	(30-j,j) Delayed or Direct	2757
LSR	D1	Format and Delayed	3217
LSR	D2	Delayed (24,6) Print	3333
LSR	D3	Delayed Decimal Integer(single)	3214.
LSR	S1	(24,6) GD Scope Column Layout	3323
LSR	S2	Single Length Scope Decimal Integer	3322
LSR	F1	(24,6) e^x	3328
LSR	F2	" x	3330
LSR	F3	" ln x	3331
LSR	F4	" sin x, cos x	3332
LSR	F5	" sinh x, cosh x	3329
LSR	F6	Arc sin x	3281

LSR	M1	Largest Eigenvalue	3168
LSR	M2	Simultaneous Equations	2624
LSR	SPL	Auxiliary Buffer	2767

144 C. Self-Consistent Molecular Orbitals are the optimum choices of linear combinations of atomic orbitals determined through a process described as a self-consistent field approximation. The numerical procedure involves matrix-vector multiplications, vector additions, and matrix diagonalization.
:for Professor J.C.Slater, Physics Department
:by Dr. A. Meckler (DDL)
:DCL Staff: Arden, 4 hours; WWI, 5 minutes

The use of different starting points indicated no difficulty with convergence of the iteration scheme. The program is therefore considered suitable for general use. However, it is planned to modify it so that the desired data can go on a separate tape and the main program can be converted once and for all independent of any particular pre-set parameters.

147 C. Energy Bands in Crystals are being studied by finding solutions of the corresponding second order linear differential equation satisfying boundary conditions at the origin. The solutions are found approximately by using the Gauss-Jackson formula for forward integration. The solutions and their first derivatives are to be combined in a sum, the weighting factors being functions of an independent parameter.
:for Professor J.C.Slater, Physics Department, DIC No. 6853
:by Dr. D.J.Howarth(DIC), 8 hours
:DCL Staff: Arden, 3 hours; WWI, 535 minutes

Production is proceeding at a most encouraging rate. The function $F(E,E_0)$ has been computed for $E = -1.8$ to $+5.0$ at intervals of 0.2, and for small values of E_0. The roots of this function are required to give a functional dependence of E_0 on E. For most E, the roots correspond to small E_0, which have now been determined. For certain values of E, however, the curve of E_0 against E has an asymptote. Hence around these E, the calculation will be carried out using a smaller range of E and using larger E_0. The results so far obtained confirm theoretical predictions of the dependence of E_0 upon E.

149 C. Digital Methods of Detecting Signal from Noise are being investigated. A sequence of binary numbers will simulate the message wherein regions of high density of ones are signal regions and those with low density of ones are noise regions. Various methods of detecting the change from one region to another, as well as the length and midpoint of the signal regions are being studied.
:for J.V.Harrington, Lincoln Laboratory
:by G.P.Dinneen, (DDL), 5 hours
:DCL Staff: Denman, 2 hours; WWI, 23 minutes

A print-out of the distribution of ones in the binary input demonstrated that the pseudo-random-number generator is working satisfactorily. A density detector was tried which actually counts the number of ones in a sixteen bit interval. This proved to be the best detector and will serve as a criterion of judgement. Emphasis is now being placed on designing a detector which changes its logic when it detects a target.

152 D. Diffusion in an Oxide-Coated Cathode is being studied to determine the effects of combined thermal and electrolytic diffusion that occur in an oxide-coated cathode when current is caused to flow through the cathode.
:for W. B. Nottingham, Physics Department, DIC No. 6345
:by H.B.Frost(Res. Assist, E.E.Dept.), 12 hours
:DCL Staff: Porter, 2 hours; WWI, 273 minutes

During the early part of this period tape 3409 was debugged completely. In this last period three one-hour runs have been completed. A partial run of $\frac{1}{2}$ hour will soon be completed. To make it easier to carry on partial runs, a routine has been incorporated into the program to punch out partial results in 556 form for re-insertion at a later date.

The 3409 form of this problem is much faster than the 3311 form, mainly due to an improved square root routine which makes use of a better first approximation. The improvement in speed is by a factor of about 6.

After each hour run, results differing from the known steady-state values by about 3 parts per 1000 were obtained. One run will be extended to check the convergence more exactly by comparing the steady-state value determined by the program with the known value. The truncation error is about 5 parts in 10,000 depending upon the parameter being run.

153 C. Gust Response of a Flexible Swept-Wing Airplane is to be determined for various values of wing loading functions, aircraft configuration and dynamic condition parameters, as input data, giving dynamic output data determining the effect of wing flexibility on gust response. The solution involves the calculation of forcing functions and the evaluation of Duhamel integrals by numerical methods. Approximately 120 pairs of linear integro-differential equations are to be solved.
:for Professor T.H.H.Pian, Aeronautical Engineering Department, DIC No.6691
:by K. Foss(DIC), 80 hours; D. Sternlight(Math.Dept), 50 hours
:DCL: WWI, 58 minutes

The design specifications of aircraft require a knowledge of the acceleration to which an airplane is subjected when flying through a gust(updraft); consequently, there has been much active research on this problem. The present problem will make a parametric study of the gust response of flexible, swept-wing airplanes using a method previously developed.

The two simultaneous integro-differential equations which are to be solved are

$$\mu_1 q_0'' + \mu_2 q_1'' + \int_0^{s_0} q_0''(\sigma)\, I^{(0)}(s_0-\sigma)\, d\sigma + \int_0^{s_0} q_1''(\sigma)\, I^{(2)}(s_0-\sigma)\, d\sigma$$

(continued on next page)

$$+a_1\int_0^{s_0} q_1'(\sigma)\, I^{(1)}(s_0-\sigma)\, d\sigma = a_0\int_0^{s_0} F'(\sigma)\, \Psi_0(s_0-\sigma)\, d\sigma \tag{1}$$

$$\mu_2 q_0'' + \mu_2 q_1'' + a_2 q_1 + \int_0^{s_0} q_0''(\sigma)\, I^{(2)}(s_0-\sigma)\, d\sigma + \int_0^{s_0} q_1''(\sigma)\, I^{(4)}(s_0-\sigma)\, d\sigma$$
$$+a_1\int_0^{s_0} q_1'(\sigma)\, I^{(3)}(s_0-\sigma)\, d\sigma = a_0\int_0^{s_0} F'(\sigma)\, \Psi_2(s_0-\sigma)\, d\sigma \tag{2}$$

where the prime mark denotes differentiation with respect to s_o, the dimensionless time-distance parameter. Since solutions are desired only for q_0'' and q_1'' (particularly $q_{0_{max}}''$) the solution will not involve numerical differention, only numerical integration.

The independent parameters of Eqs. (1) and (2) are μ_1, μ_2, a_1, and a_2; a_0 is a constant. The $I^{(n)}$ functions are known functions of s_o and δ, where δ is an additional independent parameter. The Ψ_n functions are known functions of s_o, δ, and β, where β is an independent parameter. The $F'(\sigma)$ function is a known function of s_o and s_g, where s_g is an independent parameter.

The Duhamel Integrals which occur in Eqs. (1) and (2) have the general form:

$$\int_0^{s_0} z(\sigma)\, x(s_0-\sigma)\, d\sigma \tag{3}$$

If the interval between 0 and s_o is divided into m equal stations of width ϵ, so that $s_o = m\epsilon$, the above integral can be evaluated by forming the products of z and x at successive stations and integrating them by the trapezoidal rule. Then

$$\int_0^{s_0} z(\sigma)\, x(s_0-\sigma)\, d\sigma = \epsilon\left[\tfrac{1}{2} z_0 x_m + z_1 x_{m-1} + \cdots + z_{m-1} x_1 + \tfrac{1}{2} z_m x_0\right] \tag{4}$$

From the initial conditions of the problem

$$z_0 x_m = 0$$

Thus, the Duhamel integrals can be written in matrix notation for successive values of s_o as

$$\int_0^{s_o} z(\sigma)x(s_o-\sigma)d\sigma = \epsilon \begin{bmatrix} \frac{1}{2}x_o & 0 & 0 & \cdots & 0 \\ x_1 & \frac{1}{2}x_o & 0 & \cdots & 0 \\ x_2 & x_1 & \frac{1}{2}x_o & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ x_{m-1} & x_{m-2} & x_{m-3} & \cdots & \frac{1}{2}x_o \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_m \end{Bmatrix} \tag{5}$$

Eqs. (1) and (2), therefore, can be written in matrix notation as

$$[A]\{q_o''\} + [B]\{q_1''\} = a_o\{f^{(o)}\} \tag{6}$$

$$[C]\{q_o''\} + [D]\{q_1''\} = a_o\{f^{(2)}\} \tag{7}$$

where each of the matrices [A], [B], [C], and [D] is of the same general form as Eq. (5); specifically, they are triangular matrices, each column of which is formed by shifting the preceding column down one row (only the first column need by computed to form the matrix).

For the problem at hand, there will be 20 intervals of numerical integration, each of length $s_g/10$, which yields matrices of order 20 x 20. Because of the triangular matrices involved, Eqs. (6) and (7) can be developed into relatively simple recurrence formulas for each value of q_o'' or q_1'', in terms of all previous values. There will be one or two thousand such solutions for different combinations of the independent parameters, δ, β, s_g, μ_1, μ_2, a_1, and a_2.

For each different combination of δ, β and s_g, the functions $I^{(n)}$, Ψ_n, and $F'(\sigma)$ must be recomputed. The function $F'(\sigma)$ is simply

$$F'(\sigma) = \frac{\pi}{2s_g}\sin\frac{\pi}{s_g}\sigma \tag{8}$$

The $I^{(o)}$ function is given by

$$I^{(0)}(\sigma) = \frac{\sigma^2}{32\delta} \ln\left(1 - \frac{4\delta}{\sigma+4}\right) - \frac{\delta}{4} + \frac{\sigma+4}{8} \tag{9}$$

The other $I^{(n)}$ functions are similar, each containing the same logarithmic term. The expansion of this logarithmic term about unity into a Taylor's series results in

$$I^{(n)}(\sigma) = \frac{1}{n+1}\,\frac{\sigma+2}{\sigma+4} - \frac{\delta/2}{n+2}\left\{1 + \left(\frac{\sigma}{\sigma+4}\right)^2\right\} - \frac{\delta}{2}\left(\frac{\sigma}{\sigma+4}\right)^2 \sum_{r=1}^{\infty} \frac{1}{r+n+2}\left(\frac{4\delta}{\sigma+4}\right)^r \tag{10}$$

The termination of this series at $r = 4$ has been found to give sufficient accuracy for all the values of δ and σ which will be used.

The ψ_n functions are similar but more complicated functions, and they must also be expanded into a series for numerical evaluation.

During this period programs were written and tested for evaluating the I, n, ψ, and f functions. The ψ and f functions were successfully evaluated after certain parameters had been suitably scale factored. The program for evaluating the I functions has not given satisfactory results. Further study is being carried out.

where

155 B. Synoptic Climatology. A multiple regression formula is used to predict temperatures from pressure distributions described by Tschebycheff polynomials. The matrix of scalar products which is used in the calculation of the coefficients of the multiple-regression system is being calculated on WWI.

:for Professor T.F.Malone, Meteorology Department
:by R. Miller(DIC)
:DCL Staff: Arden, 13 hours; Porter, .5 hour; WWI, 26 minutes

A (30,0) decimal integer print routine has been successfully completed for use in the program. The data for the first production run is being prepared.

157 C. Rectangular Matrix Multiplication is being programmed for inclusion in the S&EC Library of Subroutines.

:for Dr. A. Meckler, Solid State Physics
:by Dr. A. Meckler(DDL), 4 hours
:DCL: WWI, 23 minutes

The multiplication of two rectangular matrices has been successfully programmed and the routine is now included in the S&EC Library of Subroutines. The main program using this routine must plant the various orders (which should not exceed 32) of the matrices and the drum groups on which the matrices are stored. The multiplication routine then carries out the calculation in (24,6) leaving each matrix factor on its drum group and the product on another.

1.3 Operating Statistics

1.31 Computer Time

The following indicates the distribution of WWI time allocated to the S&EC Group.

Programs	53 hours, 10 minutes
Conversion	13 hours, 10 minutes
Magnetic Drum Test	19 minutes
Magnetic Tape Test	26 minutes
Scope Calibration	52 minutes
Demonstrations(#131)	37 minutes
Total Time Used	68 hours, 34 minutes
Total Time Assigned	74 hours, 12 minutes
Usable Time, Percentage	92.2%
Number of Programs	238

1.32 Program Time Distribution

The following table attempts to show how the WWI time expended on S&EC programs was distributed with respect to machine runs that gave meaningful results (productive computer time) and runs that gave unsatisfactory results ("lost" computer time). Productive computer time is subdivided to indicate the time involved in actual computations as contrasted with the time expended getting information out of WWI. Computer time "lost" is subdivided to show the portion of time lost due to errors in the programmer's formulation of his problem (logical errors); due to errors in the programmer's use of the WWI code, CS Conventions, etc. (technical errors); due to tape preparation errors; due to errors by the S&EC computer operators in running the program; due to malfunctioning of terminal equipment; and finally due to miscellaneous causes.

These times are determined as percentages of the time listed above in section 1.31 for programs. The figures below have been averaged over the last three biweekly periods. The times used in computing these figures are extracted from the biweekly report forms submitted by the various programmers who have used S&EC allocated WWI time.

1. Productive Computer Time
   - Computation 55.8%
   - Output 9.9%
2. Computer Time Lost Due to Programmers' Errors
   - Technical 19.5%
   - Logical 4.3%
3. Computer Time Lost Due to Other Difficulties
   - Tape Preparation 2.7%
   - Operator's Errors 2.0%
   - Terminal Equipment Malfunction 2.2%
   - Miscellaneous 3.6%

1.33 Tape Preparation (M. Mackey)

An attempt is being made to obtain some idea of the time expended in the preparation of tapes. During the past biweekly period a check was made on the tapes processed.

Due to the variations in procedures involved we have distinguished among original complete tapes and the following three types: typed modifications - changes of 11 or more registers which must be typed, converted, then attached to the main program or changes which must be made in the body of a Flexowriter tape; manual modifications - changes punched directly in 556 form and attached to a converted tape; combined tapes - which require duplication of two or more complete tapes.

The following information was compiled:

	Complete Tapes	Typed Mods	Manual Mods	Combined Tapes
No. of Tapes	141	55	48	79
No. of Registers	38,213	1277	247	
Time Consumed	115 hrs. 35 min.	27 hrs. 53 min.	8 hrs. 11 min.	10 hrs. 10 min.

Thus, it may be seen that the average length of an original complete tape is 271 registers requiring 49 minutes to prepare. A typed modification averages 22 registers in length and requires 29 minutes to prepare while Manual Modifications average 6 registers and require 10 minutes for preparation.

2. COMPUTER ENGINEERING

2.1 WWI System Operation

2.11 Core Memory (L.D.Holmes, A.J.Roberts)

The drum parity-check system has been installed and is now functioning. Drum parity alarms should occur only on the bi or rd orders. Core Memory alarms should not occur on these orders.

2.12 Auxiliary-Drum System (K.E.McVicar)

The parity system has been installed for the auxiliary drum, and parity checks are now made on all transfers from the drum.

An attempt is being made to revise the paperwork for the drum so that it fits into the Whirlwind system. This includes the compilation of parts lists, making of wiring schedules, and use of WWI modification notices for all drum-system changes.

2.13 Magnetic Tape (E.P.Farnsworth)

Minnesota Mining and Manufacturing has finally received a shipment of mylar from Dupont and expects that 80 percent of the tape we have on order will be shipped splice-free. They have just developed a new splicing tape having less than one-tenth the stretch of the standard 41 splicing tape; splices made with a sample of the new tape appear to be usable but may cause trouble after a few months storage or use. Three-M has also advised that tri-acetate base tape is not adequate for our application. Audio-tape is shipping us two sample reels of unspliced high-output mylar-base tape.

The adapters necessary for converting Unit 2 to triangular-hub reels were completed by the shop and are ready for installation. Panels for operating a second printout on Unit 2 are being installed. Replacement of a pair of electrolytic capacitors in the servo-chassis which caused instability with Unit 0 previously and Unit 2 presently has apparently cleared the trouble. These capacitors are being checked for intermittent leakage.

A trouble-report sheet has been prepared, and the marginal clutch test program has been modified to correct for the change in computer speed.

2.14 Typewriter and Paper Tape (L.H.Norcott)

Recent carriage-return troubles on the delayed-output printers were apparently caused by extra "start tape" pulses originating in the magnetic-tape printout-control register. Farnsworth has modified his circuits to correct this condition.

2.2 Terminal Equipment

2.21 Ferranti PETR (F.E.Irish)

Sketches for the mechanical and electronic assembly of the amplifier circuits of the Ferranti PETR and a sketch of the control circuits have been given to the drafting room.

According to the present schedule, one Ferranti PETR will be installed in a temporary fashion during the next two weeks. The construction of the final amplifier units for the three Ferranti PETR's and the control circuits for one unit will be completed by February 1, 1953. The final installation will follow that date.

2.3 General

2.31 WWI Service File (D.A. Morrison)

Plans are under way to relocate the WWI Service File in WWI Test Control to allow the Flexowriter console to be moved out of the passageway. The contents of the steel cabinets are to be consolidated into a smaller cabinet, and the WWI Service File and the smaller cabinet will occupy the space now used by the steel cabinets. The upper left-hand drawer of the desk in Test Control now contains the test tapes.

3. LIBRARY ACCESSIONS LIST

The following material has been received in the Library, W2-325.

Library Files

No.	Source	Title
2592	Business Week, Reprint	Computers Go Commercial
2593	NBS	Methods of Conjugate Gradients for Solving Linear Systems
2598	Rensselaer Computer Lab.	A Magnetic Tape Recorder for Analog Computing

4. ADMINISTRATION AND PERSONNEL

Terminated Staff (J.C.Proctor)

Richard Farmer

Terminated Non-Staff (R.A.Osborne)

Assunta Aprile
Nancy Jones
Athena Korologos
Esther Sidman