AIR TRAFFIC CONTROL PROJECT

Report R-191

STUDY OF A SYSTEM FOR FOLLOWING A FLIGHT PROGRESS SCHEDULE

Report by

Dr. William K. Linvill

SERVOMECHANISMS LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge 39, Massachusetts
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ABSTRACT

A schedule-following system for an airplane is a servomechanism. By use of conventional design techniques the system can be made to operate successfully up to the limits of the fixed part of the system, which is the airplane itself and the navigation system. For a Lockheed Constellation and an accurate (± 250 feet) navigation system, the speed control can have a bandwidth of about 0.4 cycle per minute and a speed range of about ± 15 percent of cruising speed, and the position can be controlled to within about ± 500 feet. Winds and navigational noise are the main corrupting disturbances on the system. By proper design of the control system, the effect of winds on ground speed can be diminished almost to extinction as long as the required air speed is within the range of the airplane engine. Navigation-system errors cannot be overcome by compensation.

During initial approach, use of airbrakes to provide negative thrust and use of gravity to provide additional positive thrust can increase the incremental thrust range by a factor of four for a descent of 1 foot in 10.

STUDY OF A SYSTEM FOR FOLLOWING A FLIGHT PROGRESS SCHEDULE

1. INTRODUCTION

Automatic flight control is the process of automatically guiding an airplane along a prescribed path in accordance with a prescribed time schedule. Both the path and the schedule may be changed during flight. Flight control is exercised through throttle, elevator, aileron, and rudder controls. Speed control is exercised through throttle control and is used to keep the prescribed time schedule. To a first approximation, schedule control can be thought of as independent of other controls; hence there is no loss in generality in assuming the path to be a straight line.

The objective in the design of a speed-control system is to cause a given airplane with a given navigation system to keep a schedule within a desired tolerance. The quality of the system performance is often expressed in terms of system bandwidth and limits of linear range, along with freedom of the system response from effects of corrupting disturbances. The system bandwidth gives a measure of how fast the plane can be made to follow schedule changes; the limit of linear range gives a measure of the speed changes that can be followed with smooth operation.

Assuming optimum controller design, the performance limits are set by two factors: (1) the response limitations of the airplane which are set by the peak incremental thrust available from the engine to change the speed of the plane, together with those parameters, such as inertia and drag, which determine the thrust necessary to change speed, and (2) corrupting disturbances such as headwinds and navigation noise which cause an error in position or speed. Without optimum controller design the performance limits are not obtained. How nearly they are obtained is a measure of the quality of the design.

As in any control system design, the design data for a speed controller include: (1) description of the dynamic characteristics of the output member (the airplane), usually in terms of a transfer function, (2) a statement of the limits of the linear range of the output member (limits of thrust and speed range over which the airplane's responses are describable by a transfer function), (3) an accurate description of the corrupting disturbances whose effects are to be minimized by good design.

This report emphasizes design of the servo system for speed control. Equations for the flight characteristics of an airplane have been taken from the appendix of Summary Report 4.

1.1 Assumptions made in the analysis.

In order to simplify the analysis, the following major assumptions are made.

(1) The components of velocity, thrust, drag, and acceleration along the flight path effectively equal the magnitudes of these vector quantities. This is a good assumption because the angles between these vectors and the flight path are small.

(2) Gusts, which disturb attitude momentarily but which do not last, have no long-term importance and consequently have no appreciable effect on speed control. Attitude of the aircraft can be closely controlled even in the presence of corrupting effects of gusts because attitude variables are easy to measure and are directly controlled. Consequently no long-lasting attitude disturbances exist. Usually path variations are associated with at least the first integral of an attitude variable, and path variations do not result from attitude variations which do not last for some time. In short, the plane is a low-pass filter, and it is not sensitive to gusts, which are high-frequency disturbing signals.

(3) The only winds of importance are headwinds or tail winds, or the components of cross winds along the path. Those components normal to the path are corrected for by lateral path control (by establishing a crab angle). The most important wind to overcome is a high head wind or tail wind, because it exerts peak thrust along the path.

(4) Linearized analysis and incremental values should be used because drag is a non-linear function of speed. Linear approximations are acceptable as long as the range of increments is not large.

1.2 Approximate characterization of the thrust-speed relation for the aircraft.

The relation between thrust and displacement of the craft along the path is compactly described in terms of the block diagram shown in Figure 1a.

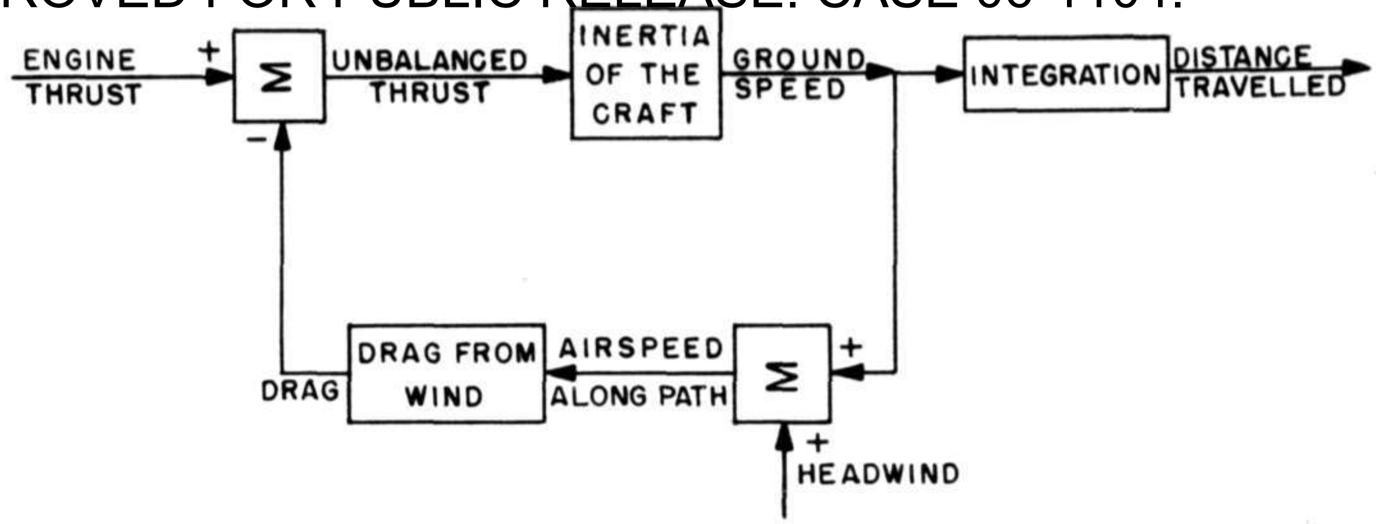

FIG. 1a. RELATIONSHIP BETWEEN THRUST AND POSITION.

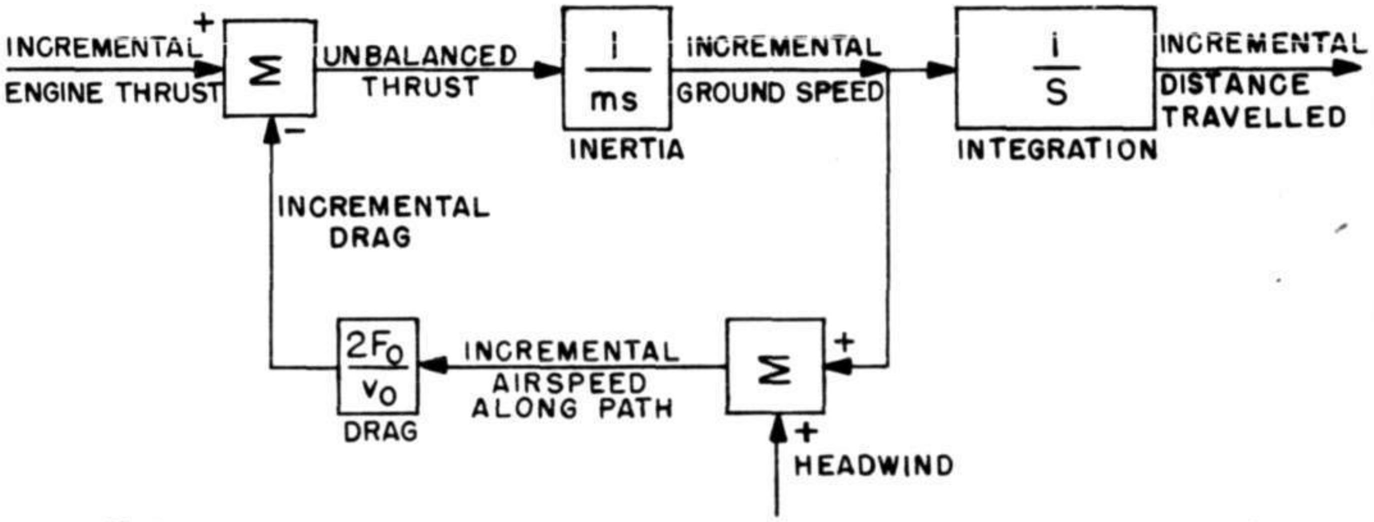

FIG 1b. RELATIONSHIP BETWEEN INCREMENTAL THRUST AND INCREMENTAL DISTANCE TRAVELLED. USE OF INCREMENTAL VALUES MAKES POSSIBLE THE USE OF TRANSFER FUNCTIONS.

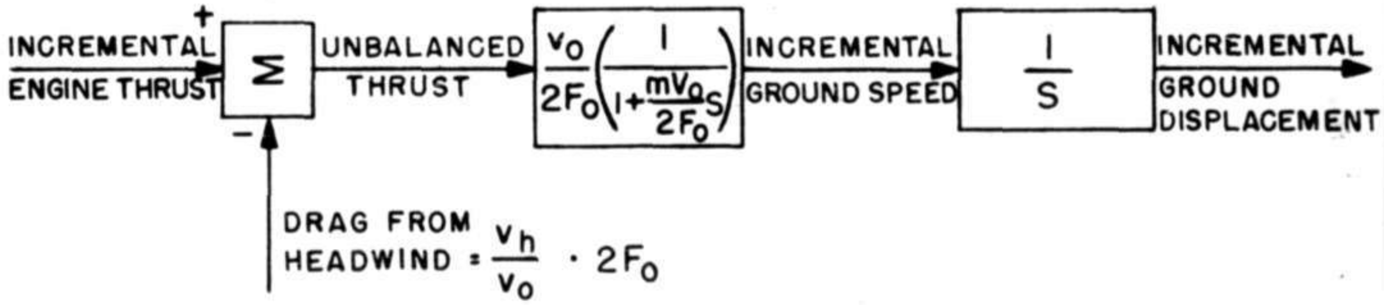

FIG 1c. SIMPLIFIED BLOCK DIAGRAM TO RELATE INCREMENTAL ENGINE THRUST TO INCREMENTAL GROUND DISPLACEMENT.

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Since the mass of the aircraft is known, the acceleration can be related to unbalanced thrust, and ground speed is the integral of the acceleration. Ground speed plus wind gives airspeed, which determines the drag. Distance travelled is the time integral of ground speed. The only complication in describing this system by transfer functions arises from the non-linearity in the drag-airspeed relation. A conventional approximation is applied in this instance. Incremental values of all the quantities can be approximately related by linear relationships. Assume that the aircraft is flying at ground speed v_o in still air with a thrust of F_o and that the drag just equals F_o. These conditions are called quiescent conditions. An approximately linear relation exists between incremental drag and incremental airspeed.

If F_d = drag at any airspeed v, then

$F_d = F_o(\frac{v}{v_o})^2$, since drag varies as the square of the speed.

Let $F_d = F_o + \Delta F_d$

and $v = v_o + \Delta v$.

$$\Delta F_d = F_o\left[\frac{2\Delta v}{v_o} + (\frac{\Delta v}{v_o})^2\right].$$

If $\Delta v << v_o$,

$$\Delta F_d \approx \frac{2F_o}{v_o}\Delta v.$$

Since the unbalanced thrust under quiescent conditions is zero, the incremental unbalanced thrust equals the total value. The incremental unbalanced thrust equals the mass, m, of the aircraft times the rate of change of the ground speed, which equals the rate of change of incremental ground speed. The block diagram of Fig. 1a can be modified to use incremental values, giving 1b.

The head-wind disturbance can be related to the resulting drag and referred to the thrust summing point. It is then possible to find how the combined incremental engine thrust and head-wind drag is related to the incremental ground speed. Call the Laplace transform of combined incremental engine thrust and head-wind drag F, and call the Laplace transform of incremental ground speed V. The complex-frequency variable used in the Laplace transform is s.

Then

$$\left(F - \frac{2F_o}{v_o} V\right) = msV.$$

or

$$F = msV + \frac{2F_o}{v_o} V$$

The ratio $\frac{V}{F} = \frac{v_o}{2F_o} \left(\frac{1}{1 + \frac{mv_o}{2F_o} s} \right)$. With this relationship the feedback loop of Figure 1b may be eliminated, and the simplified block diagram of Figure 1c results.

If position error were used to actuate the thrust control (throttle) the block diagram of the overall system could be represented by Figure 2. The transfer function of the engine will be assumed to be a constant K_e and to have no time lag. Actually this assumption is optimistic, especially for jets. The engine response is characterized by a transfer function of at least 1 time lag. To see whether the engine time lag is important, compare its characteristic time with the time constant $\frac{mv_o}{2F_o}$ of the airplane. For a 4-engine propellor-driven plane, F_o is of the order of 7000 pounds, v_o is of the order of 350 feet per second, and mass is of the order of 3000 slugs. The time constant of the airplane then is about 75 seconds.

Without compensation, the speed-control system's feed-forward section has a transfer function of $\frac{K_e v_o}{2F_o} \frac{1}{s(1 + \frac{mv_o}{2F_o} s)}$. Call $\frac{K_e v_o}{2F_o} = K$ and $\frac{mv_o}{2F_o} = \tau_a$. The overall system function of the uncompensated system of Figure 2 would be

$$\frac{\frac{K}{s(1 + \tau_a s)}}{\frac{K}{s(1 + \tau_a s)} + 1} = \frac{K}{\tau_a s^2 + s + K}$$

The simple system of Figure 2 is very inadequate. By merely changing the system gain, it is not possible to improve the frequency characteristics of the system. With a low value of K, the system has a low cutoff frequency (narrow bandwidth). If K is increased to increase the bandwidth, the system becomes very much underdamped. One system

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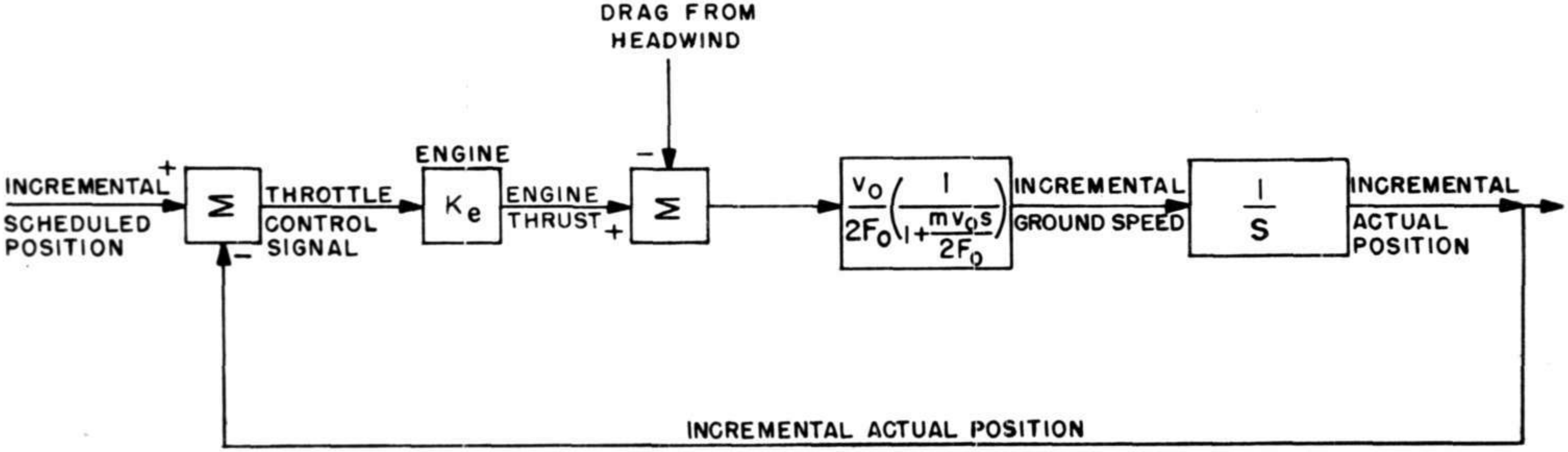

FIG. 2
BLOCK DIAGRAM OF OVERALL SYSTEM WITHOUT COMPENSATION.

design objective should be to increase the bandwidth without decreasing the damping. There are three standard types of compensation to achieve this objective: (1) a matching lead controller, (2) airspeed feedback, (3) ground-speed feedback.

If the objective of the use of compensation were only to modify the overall system frequency characteristics, all three types of compensation would be equally useful. In addition to modifying the frequency characteristics, it is important to minimize the effect of corrupting disturbances whose effects should be minimized are head winds and corruptions from the non-linear drag-speed relation. Both airspeed feedback and ground-speed feedback minimize corruptions from the non-linear drag-speed relation. If ideal ground-speed feedback were available, it would be most acceptable because directly it would minimize corruptions from headwinds. Airspeed feedback does not do this because it does not separate ground-speed and wind.

The matching lead controller is effectively a high-pass filter which would operate on the throttle control signal (Figure 2). It is unsatisfactory because it is placed at a spot on the loop where all high-frequency corrupting signals are largest.

The practical choice, then, is between airspeed and ground-speed feedback. Actually there exists no simple way of measuring ground speed directly. It could be measured indirectly by putting a lead controller in the feedback loop to obtain an approximate derivative of the position signal. This technique is not satisfactory because there is a large amount of high-frequency noise present in the navigation signals which indicate position. With present equipment, airspeed feedback is most practical, and though it does not minimize corruptions from headwinds directly, it allows the overall system to be stabilized with a high loop gain. The high gain reduces corruption from headwinds. Thus corruption effects of headwinds are diminished indirectly by airspeed feedback.

The overall system with airspeed feedback compensation can be represented by Figure 3. The airspeed feedback is represented as ground-speed feedback plus a corrupting signal equal to velocity. The corrupting signal is positive for a headwind, negative for a tailwind.

1.3 Possible Corrupting Signals.

To evaluate the closeness of schedule following possible with the system shown in Figure 3 and to have proper perspective in further design of compensation network, the designer must attempt to characterize all the possible corrupting disturbances, though the characterization may be inexact. Figure 4 is a table listing and describing the important corruptions appearing in the speed control system.

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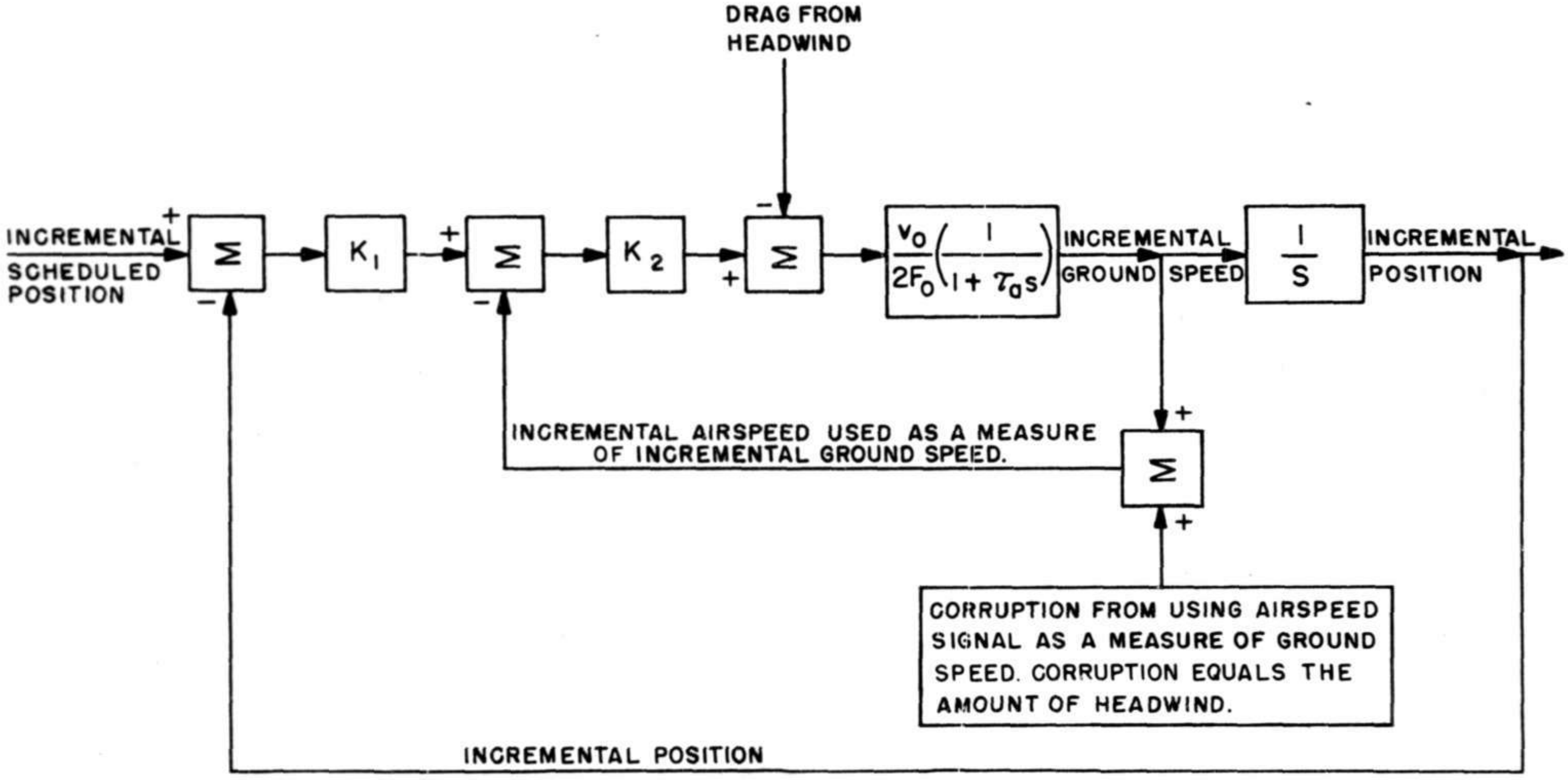

FIG. 3
OVERALL SPEED CONTROL SYSTEM WITH AIRSPEED FEEDBACK.

CORRUPTION	WHERE IT APPEARS IN THE SYSTEM (Fig. 3)	SIGNIFICANT CHARACTERISTICS
1. Drag from headwind	A thrust to oppose engine thrust	Very low frequency. Independent of all other conditions. Drag from headwind whose velocity is 20% of v_o would be greater than 40% of F_o.
2. Corruption from using airspeed as a measure of ground speed.	At the airspeed measuring device.	Very low frequency. Independent of all conditions except wind. A headwind of 20% of v_o would introduce a negative corrupting signal at the airspeed measuring device of 20% of v_o.
3. Corruption from navigation data which measures position.	At error measuring device.	Any components of high frequency can be filtered out before the signal is fed back without important lags in the position data. Lack of resolution in the navigation system is very objectionable and limits the overall system.
4. Corrupting signal in drag resulting from non-linear drag-speed relation.	A thrust to add with or subtract from engine thrust.	Associated with airspeed. Never over 5 percent of quiescent thrust if airspeed stays within 20% of quiescent value. Airspeed feedback decreases the effect of this corruption.

Figure 4. Table of Corruptions.

The evaluation of the effect on the first three corruptions listed is very simple because they can all be referred to the input of system directly. There they can all be evaluated on the basis of an equivalent error in the scheduled position as shown in Figure 5.

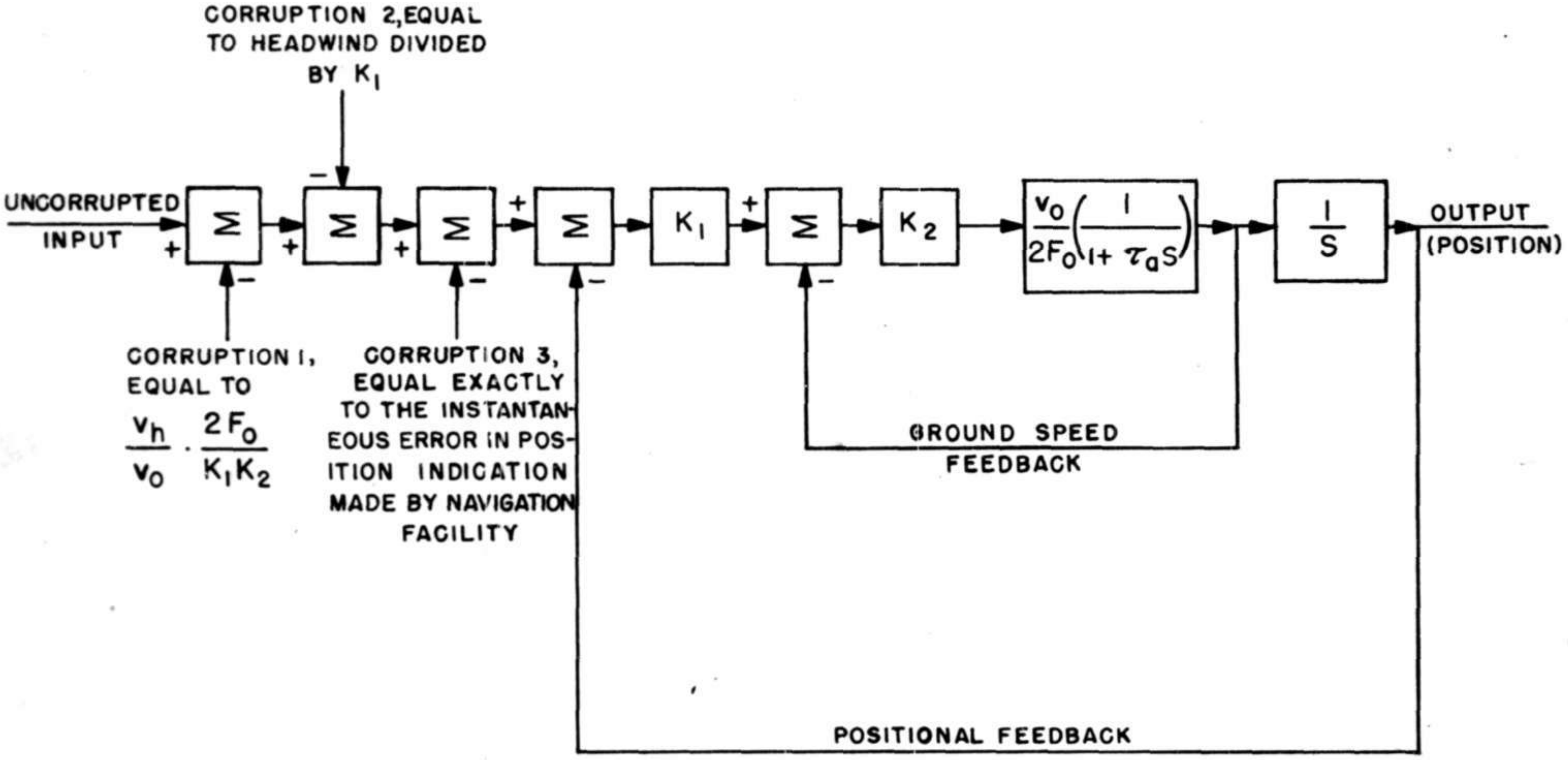

FIG. 5

CORRUPTIONS 1, 2, AND 3 REFERRED TO THE INPUT.

Clearly, increasing gains K_1 and K_2 decreases the effect on the signal from corruption 1. Increasing K_1 decreases the effect on the signal from corruption 2. Nothing can diminish effect on the signal from corruption 3 because corruption 3 is always compared directly with the scheduled position. This fact demonstrates the truth of the statement that a servo system cannot control a variable more accurately than it can be measured.

If the frequency characteristics of the corrupting signals are different from characteristics of the scheduled position, which is the true input, corrective networks can be used which minimize the output from these corruptions while not disturbing the faithfulness of the response to the scheduled position input. At this time not enough data on the nature of the corruptions is available for such elaborate design. Corrupting signal 4 is a proportional-type corruption the effects of which can be minimized by increasing the loop gain. Both airspeed feedback and position feedback are helpful in this regard.

1.4 Linear range.

The discussion of corruption given above is valid only when the system is operating in the linear range. If a given collection of inputs calls for a higher thrust than the engine can provide, then the linear representation provided by this simple analysis is no longer accurate. The limits of linear range of the system must be described in order to establish limits to closeness of following.

The quantity in the system which will be limited by saturation is engine thrust. Peak engine thrust cannot exceed the quiescent value by more than about 30%. Engine thrust can be cut to zero but can't be made negative. The limitation in engine thrust limits the speed range, the acceleration range, and the range of winds which can be corrected for by the positional feed-back.

2. DESIGN OF A SPEED CONTROL SYSTEM TO MEET SPECIFIC OBJECTIVES.

To obtain an adequate picture of the design procedure used to make the system both insensitive to corruptions and rapid in response to schedule changes, specific problems will be discussed. Though exact data are not available for a thoroughgoing design, the following simple examples are given to indicate how the information of Section 1 can be applied in specific design problems.

2.1 Allowable schedule variations

In this section it is assumed that there is no head wind, no navigation error, and no corruption from non-linear effects of drag. Effectively the question asked of the designer here is what sort of system function will give acceptable responses and what sort of schedule changes can be followed by the aircraft when saturation is the only limitation. To answer this question, he must find the allowable system bandwidth and the linear range.

As is usually true when a system has limited output thrust, an increase in bandwidth means a reduction of linear range, and an increase in linear range can be obtained only by decreasing bandwidth or by increasing the available thrust.

The greatest allowable bandwidth which can be used in the overall system functions can be deduced from a study of the limits of the output member (the aircraft). The relation between the incremental engine thrust and the output displacement as a function of frequency is given by Eq. 1.

$$\frac{v_o}{2F_o} \quad \frac{1}{1+\tau_a s} \cdot \frac{1}{s} = \frac{\theta_o}{F}, \qquad (1)$$

where $\frac{\theta_o}{F}$ is the ratio of the vector representing incremental position to the vector representing incremental engine thrust for steady-state operation at complex frequency s.

τ_a and $\frac{v_o}{2F_o}$ are constants of the airplane described in section 1.2. θ_o is measured in feet; F is measured in pounds.

For sinusoidal steady-state motion at frequency f cycles per second or $\omega = 2\pi f$ radians per second, s will be equal to $j\omega$, where j is the imaginary operator. The period, p seconds, of such a sinusoid is defined by Eq. 2.

$$p = \frac{1}{f} = \frac{2\pi}{\omega} \qquad (2)$$

Using the constants for the Lockheed Constellation, a typical 4-engined transport plane, one can relate the amplitude of the sinusoidal output to the period of the sinusoid when the incremental thrust is fixed at a peak value of $0.3F_o$.

$$|\theta_o| = \frac{1}{40} \cdot \frac{p}{2\pi} \frac{2100}{\sqrt{1+\left(\frac{150\pi}{p}\right)}} = \frac{8.37p}{\sqrt{1+\left(\frac{471}{p}\right)^2}}. \qquad (3)$$

The curve of sinusoidal output as a function of frequency for saturation thrust is plotted in Figure 6.

The response of the system (the actual position of the aircraft) to input scheduled position is measured by the navigation system. The accuracy of most navigation systems is no better than ± 250 feet. From Figure 6 it is evident that saturation thrust cannot produce this response for signals whose period is less than 130 seconds. Consequently, it is unwise to have a system bandwidth of greater than the order of 1/130 cycle per second unless one has available more saturation thrust. This is true because at frequencies greater than 1/130 cycle per second the peak output cannot be detected by the navigation systems. Whether it is feasible to use a bandwidth as great at 1/130 cycle per second or not depends upon the quality of the navigation data. To characterize the navigation data by an uncertainty band of ± 250 feet is a very inexact characterization. Hence, the bandwidth to be used in the example which follows is 0.02 radian per second, a very conservative value. With more exact information on the characteristics of navigation data, less conservative design of the system would be proper.

In the acceleration-limited range, Figure 6, the saturation amplitude of oscillation is proportional to the square of the period. As a result, reduction of the amplitude by some factor, K, will reduce the corresponding period by only $\sqrt{K}$ or increase the corresponding frequency by $\frac{1}{\sqrt{K}}$. If the navigation system were improved by a factor of 2 so that its errors were ± 125 feet (instead of ± 250 feet), the practical bandwidth for which the system should be designed would increase by a factor of only 1.4.

For the Constellation, which was used as a typical aircraft, the saturation speed is about 15 percent greater than the cruising speed, and the saturation thrust is about 30 percent greater than the thrust at cruising speed. These values are approximately true for all conventional transport aircraft. The time constant τ_a associated with change in speed is proportional to the mass times the cruising speed divided by the cruising thrust. For a very "clean" airplane, the value of τ_a would be higher than 75 seconds. These quantities determine the curve in Figure 6, and from this curve one can get rough-and-ready answers as to how fast schedules can be changed. For example, a sinusoidal variation of schedule of amplitude ± 1 mile would have a minimum period of about 750 seconds. In 1/2 of 750 seconds the incremental position could change from 1 mile behind schedule to 1 mile ahead, as shown in Figure 7.

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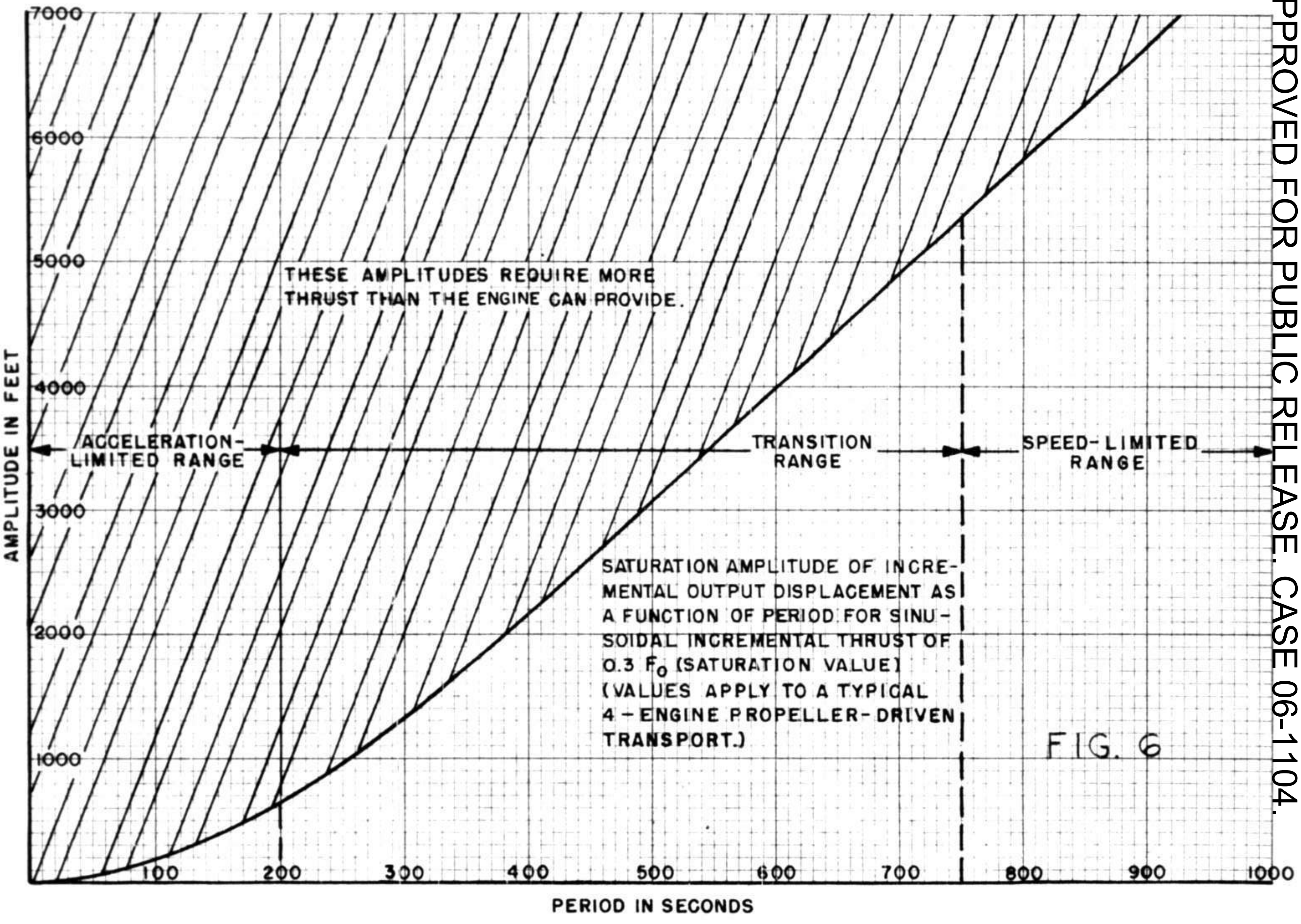

THESE AMPLITUDES REQUIRE MORE THRUST THAN THE ENGINE CAN PROVIDE.

SATURATION AMPLITUDE OF INCREMENTAL OUTPUT DISPLACEMENT AS A FUNCTION OF PERIOD FOR SINUSOIDAL INCREMENTAL THRUST OF 0.3 F_0 (SATURATION VALUE) (VALUES APPLY TO A TYPICAL 4-ENGINE PROPELLER-DRIVEN TRANSPORT.)

FIG. 6

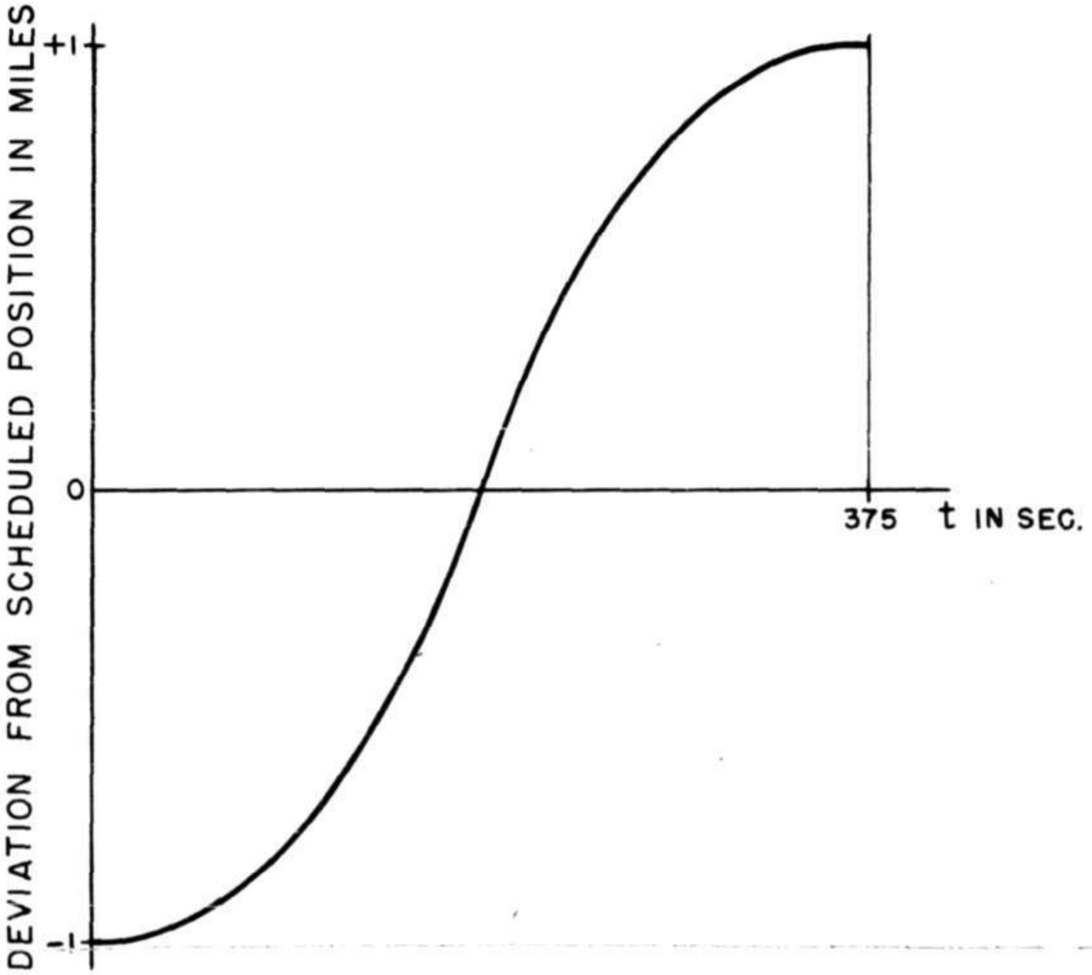

FIG. 7

INCREMENTAL SPEED CHANGES WHICH CAN BE MADE AT THE BOUNDARY OF THE SPEED-LIMITED RANGE.

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A period of 750 seconds is roughly the boundary of the speed-limited range for aircraft having τ_a's of the order of 75 seconds, and it is not feasible to attempt to change the schedule in less than 375 seconds because too large a fraction of this time would be spent in accelerating the mass rather than in operating at full speed. For periods longer than 750 seconds the amount of schedule change which can be followed is found by multiplying $.15v_0$ times the time allowed. The assumption involved in this procedure is that the time needed to accelerate to full speed is a negligible fraction of the total time used for the schedule change.

Peak engine performance has been used to select two frequencies: (1) the upper frequency limit of the system bandwidth, and (2) the highest frequency at which it would be wise to schedule a large-amplitude oscillation if the maximum amplitude limit were to be set by the top speed of the aircraft rather than by its peak acceleration. The bandwidth of about 0.02 radian per second corresponds roughly to the frequency above which the engine cannot produce a change in position great enough to be measured satisfactorily by the navigation system.

The shortest period of large-amplitude oscillation which it would be reasonable to schedule is 750 seconds. The value 750 seconds is significant because it is the boundary of the speed-limited range (Figure 6). Suppose the schedule consisted of an oscillation of period P superposed on a constant-speed schedule. For values of P greater than 750 seconds, the saturation amplitudes of this oscillation would be proportional to P. For values of P less than 750, the amplitudes would be reduced more than proportionately because too much of the engine power would be used in accelerating and decelerating the mass of the plane rather than in overcoming drag.

The two frequencies have significance in terms of the way each describes the plane's dynamic behavior. Since the system bandwidth of 0.02 radian per second corresponds to periods of approximately 300 seconds and characteristic times in the order of 100 seconds, small-amplitude transient disturbances will die out in the order of 100 seconds, though the system would not have power enough to make it wise to schedule large-amplitude incremental position changes of period less than 750 seconds.

2.2 Design to minimize corruption of the schedule by head winds.

Aside from establishing the types of schedule changes which can be followed in the absence of corruptions, one must design the system so that it is as independent as possible of the corruptions which are always present in some degree.

The system should be kept relatively free of corruption from headwinds. Effectively, a headwind results in the two corruptions of the system performance shown in Figure 5 as corruption 1 (headwind drag) and corruption 2 (error from using airspeed as a measure of ground speed). Both these corruptions are referred to the input, where they act as if they were erroneous schedule data. In Figure 5 there are no filters or elements with frequency-dependent behavior between the error and the actual application of the corrupting signal, and as a result it is very simple to refer the corruptions to the input.

2.21 Minimizing corruption on the system of Figure 5

In the simply designed system of Figure 5 the size of corruption 1 was decreased by increasing K_1 and K_2. Only K_1 reduces corruption 2. Increasing K_1 and K_2 to a reasonable point is useful. In any case, it is unnecessary to attempt to decrease the size of corruptions 1 and 2 far below the size of corruption 3 (errors in navigation system).

In the previous section, the bandwidth was set by the limitations of aircraft acceleration and navigation system accuracy. Unless a more complicated system of corrective networks than that of Figure 3 and Figure 5 is used, the selection of the bandwidth and damping defines the values of K_1 and K_2.

Let us now see how bad corruption will be with the K_1 and K_2 given by bandwidth and damping requirements. First the size of K_1 and K_2 must be calculated. The system function of the system of Figure 3 and Figure 5 is:

$$\frac{\theta_o}{\theta_i} = \frac{1}{1 + \frac{s}{aK_1} + \frac{\tau_b}{aK_1} s^2} , \qquad (4)$$

where $\frac{\theta_o}{\theta_i}$ = ratio of Laplace transform of incremental output position to the transform of the incremental input scheduled position.

s = the complex variable frequency in the transform domain.

K_1 = gain of the system connecting position error to the speed feedback summing point.

$$\tau_b = \frac{\tau_a}{1 + \frac{v_o K_2}{2F_o}} \qquad (5)$$

involved, a simplified set of modifications of the system design will be made with the purpose of reducing the corruptions without impairing the system response. The objective of the system design is to maintain (1) the bandwidth of the overall system at about 0.02 radian per second, (2) a high loop gain, (3) good damping, and, at the same time, to reduce the corruption due to head winds. In the discussion which follows the design is carried out more or less step-by-step. Effectively the problem is to reduce the corruption of the schedule by head winds without adversely affecting the overall system responses.

The two corrupting disturbances resulting from wind are corruption 1 and 2. By putting filters in the system so that the corruptions referred to the input are minimized without changing appreciably the overall system function, it is possible to make the system much better. Because the aircraft is essentially a low-pass filter, the reduction of low-frequency corruptions is more important than the reduction of high-frequency corruptions.

The size of corruption 2 referred to the input is reduced by putting a high-pass filter after the airspeed measuring device to cut down the low-frequency components of the corrupting signals. The low-frequency components of the ground speed, which are also attenuated by the filter, are not needed to adjust the overall system frequency response and damping.

A high-pass filter in the air-speed feedback channel reduces the low-frequency components of corruption 2 without disturbing the high-frequency stabilizing signals.

Inside the air-speed loop, Figure 3, the transfer function is

$$\frac{K_2 v_o}{2F_o} \frac{1}{1 + \tau a s} .$$

With the integration ($\frac{1}{s}$) cascaded, the phase shift of the overall feed-forward section at frequencies greater than $\omega = \frac{1}{\tau a}$ is greater than 135^o, and increasing the gain reduces the overall system damping as indicated by a sharp peak in the system response to high frequencies. Use of ground-speed feedback modifies this system function to make the new frequency of 135^o phase shift $= \frac{1}{\tau b}$. The effective bandwidth of the feed-forward section would not be changed by inserting in the feedback loop a high-pass filter with a gain of 1 for frequencies of $\frac{1}{\tau a}$ and higher, but with much less gain at lower frequencies. A suitable filter would be one with a transfer function $\frac{0.1 + 75s}{1.0 + 75s}$.

The new inner loop now has the configuration shown in Figure 8.

$$a = \frac{\frac{v_0K_2}{2F_0}}{1 + \frac{v_0K_2}{2F_0}}, \qquad (6)$$

The solution of the characteristic equation of the system gives roots at

$$s = \frac{1}{2\tau_b}(-1 \pm \sqrt{1-4\tau_b\, aK_1}).$$

The bandwidth of the system fixes τ_b. For a bandwidth of 0.02 radian per second the value of $\frac{1}{2\tau_b}$, the frequency at which the response is half the value at zero frequency, should be set at about 0.02. The quantity $2\tau_b$ is fixed at 50 seconds by this procedure. For a well damped system, the damping ratio, $4\tau_b\, aK_1$, should be set at 1.0.

If

$$4\tau_b\, a\, K_1 = 1. \qquad (7)$$

From the bandwidth setting,

$$\tau_b = 25. \qquad (8)$$

From Eq. 5,

$$K_2 = 80. \qquad (9)$$

From Eq. 6,

$$a = 2/3. \qquad (10)$$

From Eq. 7,

$$K_1 = \frac{3}{200}. \qquad (11)$$

Using these values of K_1 and K_2, the corruption of the input resulting from a steady headwind of .10 v_0 is:

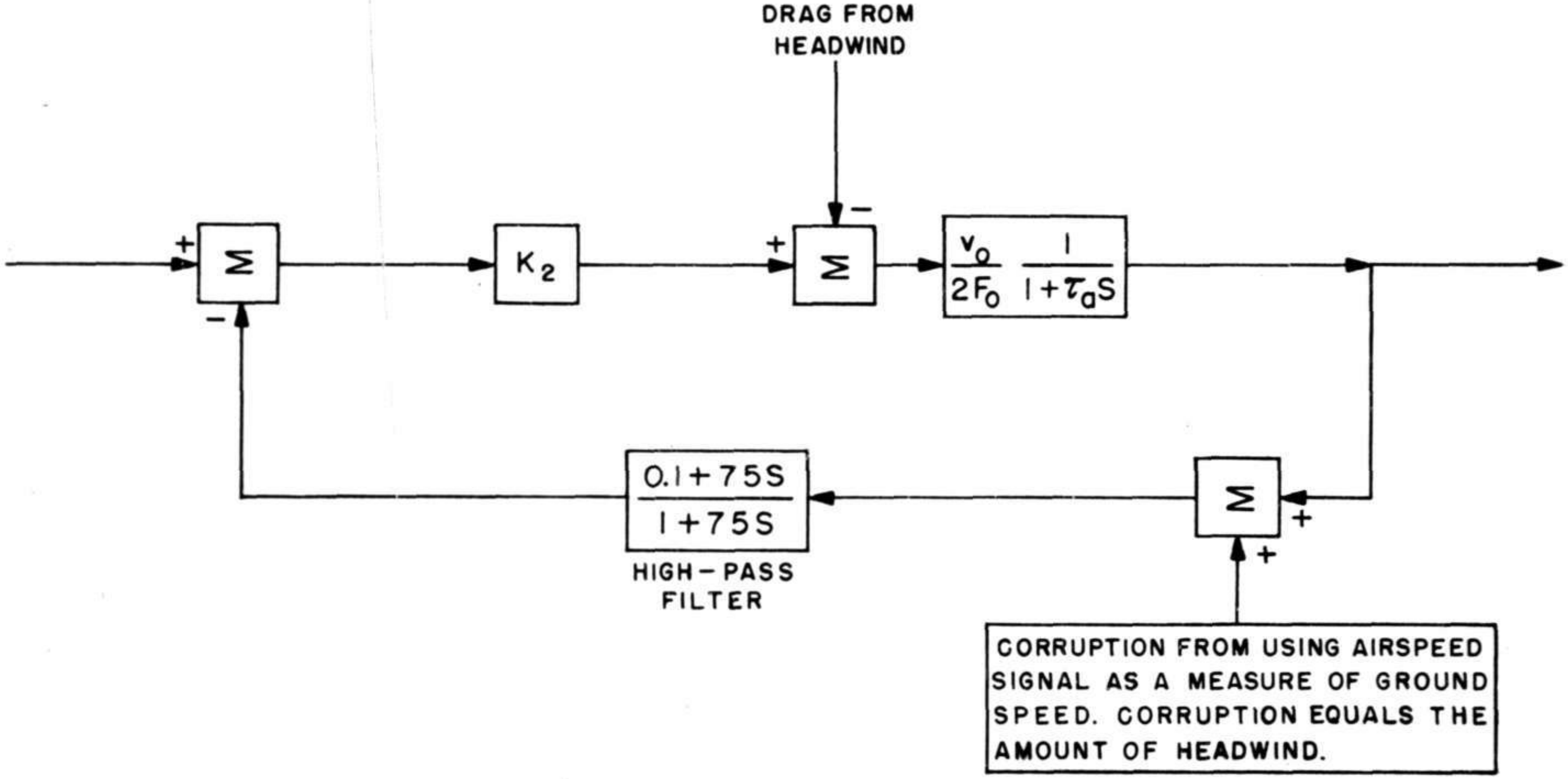
$$\text{Corruption 1 (Fig. 5)} = \frac{v_h}{v_0} \cdot \frac{2F_0}{K_1K_2} = 0.1 \cdot \frac{2 \cdot 7000}{3/200 \cdot 80} = 1167 \text{ ft.}$$

$$\text{Corruption 2 (Fig. 5)} = \frac{v_h}{K_1} = \frac{35}{3/200} = 2333 \text{ ft.}$$

Essentially, these figures indicate that a steady headwind of 10 percent of cruising speed would result in the plane's being about 3500 feet behind schedule in the steady state. Both these corruptions are large because the dynamic characteristics of the system were chosen to provide adequate damping but with no attention to reducing the effective corruption.

2.21 Selection of new corrective networks to reduce the effect of headwind without spoiling dynamic response of the system.

A more careful design could be completely carried out only after detailed description of the corruptions. To indicate the procedures



CORRUPTION FROM USING AIRSPEED SIGNAL AS A MEASURE OF GROUND SPEED. CORRUPTION EQUALS THE AMOUNT OF HEADWIND.

FIG.8

ADDITION OF A HIGH PASS FILTER TO THE INNER LOOP OF FIG.3 TO DIMINISH EFFECT OF CORRUPTION FROM HEADWINDS.

The size of K_1 can be increased to diminish further the effect of corruption 2 and also to diminish the effect of corruption 1. However, increasing K_1 over all frequencies decreases the system damping, as was discussed earlier. Increasing K_1 at low frequencies only does not affect the system damping. Hence the insertion of a low-pass filter as shown in Figure 9 will increase the gain at low frequencies without increasing it at high frequencies.

Neither the high-pass filter in the airspeed feedback loop nor the low-pass filter in the overall feedforward section will reduce corruption from high-frequency wind changes. This fact is not a shortcoming, because though the schedule is corrupted by these high-frequency disturbances the system bandwidth is not wide enough to allow the actual output position to be greatly corrupted. An overall system with its effective corruptions given in terms of their Laplace transforms is shown in Figure 9.

The overall system of Figure 9 has a very simple low-pass filter after the error detector. If used as it now stands, the system would be somewhat underdamped. This underdamping can be modified by proper compensating networks. To carry out these details is not necessary or feasible now. Exact system transfer functions of the aircraft are not available, and the example was carried this far merely to indicate that the procedure described is workable.

Summarizing, the study described above indicates that the following results can be obtained in the design of a speed control system of a typical 4-engined transport plane with a cruising speed of 350 feet per second (240 miles per hour):

1. The overall system bandwidth can be made to be about 0.02 radians per second (about 0.4 cycle per minute).

2. The main corruptions which affect the system schedule adversely are:

   (a) A schedule corruption of about ± 250 feet error from navigation noise.

   (b) A corruption from the use of airspeed feedback in the presence of winds, which in the steady state would corrupt the schedule by a number of feet approximately equal to the velocity of the head winds in miles per hour. (A 60-mph head wind would result in a corruption of this type of about 60 feet.)

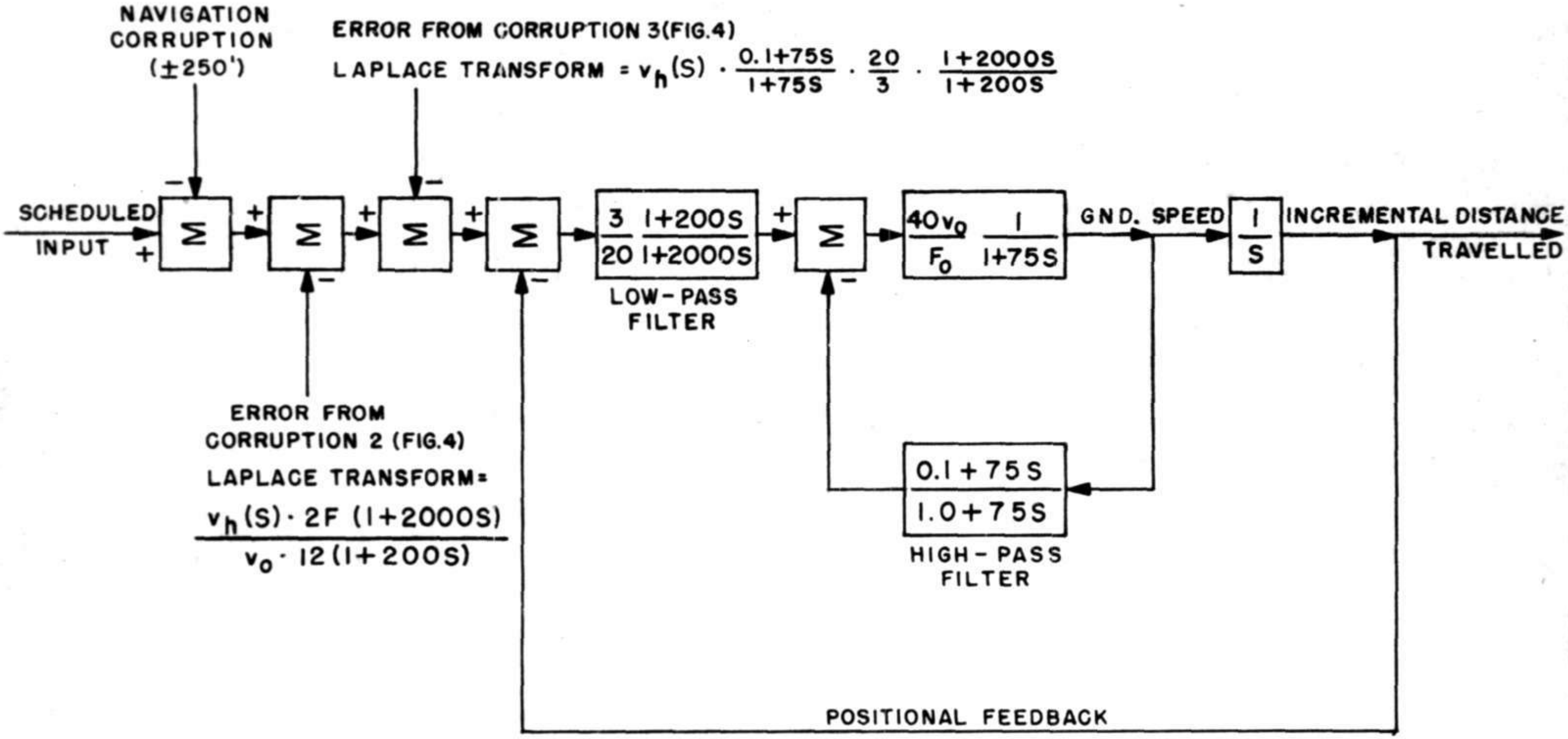


FIG 9

IMPROVED OVERALL SYSTEM DESIGNED TO REDUCE CORRUPTION

(c) A schedule corruption from drag from head winds. With steady head winds (or tail winds) this corruption in feet is 5 times the wind velocity in miles per hour.

We see that building corrective networks specifically to reduce corruption from steady winds allows reduction of these corruptions by a factor of 10. Fast changes in head winds (gusts) are not usually important because they do not come into frequency ranges in which the overall system bandwidth (0.02 radians per second) allows substantial output response to the corruptions. It is possible for transient corruptions to be 10 times as great as the steady-state values. Even then, when the system is operating in its linear range, actual transient position errors of 1000 feet would be extremely rare.

The values presented above are valid only when the system is operating in its linear range; i.e., only when the engine can produce the thrust called for by the control system. For head winds greater than about 15 percent of cruising speed the engine can not produce enough additional thrust to overcome the increased drag, and the plane will be blown off schedule.

2.3 Closeness to which a schedule can be maintained during initial approach.

There are three important factors which distinguish initial approach procedures from inter-airport flight:

(1) Fast change in altitude results in fast changes in wind velocity.

(2) If the schedule position chosen when the control system is turned on results in a large initial error there may not be time enough to correct the error during the entire initial approach.

(3) The use of air brakes makes possible the extension of the available incremental thrust by about a factor of 3 or 4.

Factors 1 and 2 indicate that conditions peculiar to initial approach make it desirable to have increased linear range of the control system during initial approach to insure that a schedule can be kept.

Factor 3 indicates that air brakes are a practical means for increasing the linear range. Unfortunately brakes can be used only for producing negative thrust. High positive thrust is available during descent because of the component of gravitational pull along the approach path. Brakes can be used to get positive incremental thrust by applying them steadily during descent and by removing them when a positive increment is called for. Reports on experimental work on air brakes indicate that use of brakes at high speed can produce a negative thrust about equal to the weight of the aircraft.

Probably descent speeds can be controlled readily over a range of 0.4 v_o to 1.1 v_o. If the average approach speed is chosen as 0.75 v_o, the incremental limit of speed is 0.35 v_o. The incremental thrust can be roughly F_o for a descent slope of 1 to 10.

In order to get the negative incremental thrust of F_o, there must be a steady-state positive thrust of F_o. At a negative incremental thrust of F_o the resultant thrust must be zero, or the gravitational pull must be opposed by brakes.

The behavior of air brakes is decidedly non-linear, and any attempt at an analysis from the limited information we now have is fruitless. It is quite probable that usual linearizing results of feedback would be effective in making the overall system more nearly linear. For the discussion here it is assumed that application of airbrakes is usable on the same control system used for level flight, and that it can increase the linear range by a factor of four.

All overall system responses have a characteristic time in the order of 100 seconds as long as the system operates in its linear range. Comparison of the approach system with the system of Figure 6 and Figure 7 shows that on approach a change from 4 miles behind schedule to 4 miles ahead could be brought about in 6 minutes. Headwinds (or tail winds) of 0.35 v_o could not blow the plane off schedule. From the system in Figure 9, it can be shown that a positional error of less than 1000 feet will call for full incremental correcting torque.

The possibility of using air brakes as a means of increasing the linear range of the system is striking. However, the use of air brakes during descent requires a steeper angle of descent, and consequently a longer flight before descent is started. This will require some additional fuel consumption. (It is generally true in control systems that additional linear range requires additional power.) The additional fuel consumed must be weighted against the fuel (and time) which would be expanded in a holding pattern if precise scheduling were not possible.

Signed by: William K Linvill
Dr. Linvill

Approved by:
Jay W. Forrester

WKL:bjm

Drawings:

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