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SUBJECT: SELECTION SYSTEMS FOR MAGNETIC CORE STORAGE

To: William N. Papian

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Abstract: Several core-selecting systems can be devised that offer better selection ratios than the standard 3-dimensional array. Improved selection ratios result in reduced storage access time but at the cost of considerably increased complexity of the driver circuits.

INTRODUCTION

A storage system using a 3-dimensional array of magnetic cores has been under study in the laboratory for some time.[1,2] It is assumed that the reader is familiar with the system as described in the above references.

Very promising progress has been made, especially recently. It is still essentially true that neither the steel nor the ceramic cores now available present a satisfactory solution to the storage problem:

a. The steel cores have the proper rectangularity but switch too slowly.

b. The ceramic cores switch rapidly but are not sufficiently rectangular.

Both situations can be improved if the ratio of selecting to non-selecting H's can be increased.

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1. R-187, "Digital Information Storage in Three Dimensions Using Magnetic Cores" by J. W. Forrester.

2. R-192, "A Coincident Current Magnetic Memory Unit" by W. N. Papian

a. For the steel cores the non-selecting H can remain as is and the selecting H increased to decrease the switching time.

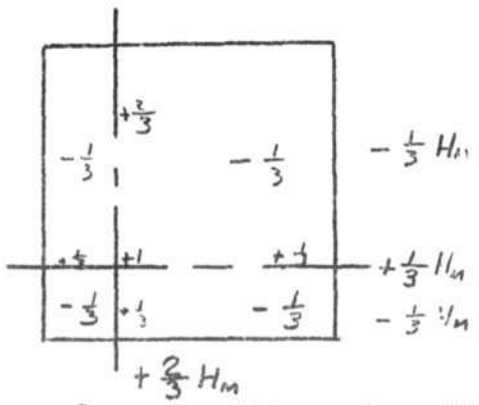
b. For the ceramic cores the switching H can remain as is and the non-selecting H reduced to improve signal-noise ratio, etc.

The switching system described by JWF is simple, elegant and "best possible" 3-dimensional in a sense to be defined later. Nonetheless it appears worthwhile to consider switching systems that result in improved selecting ratios even though they may result in more selecting equipment.

A 2-Dimensional System with a 3:1 Selecting Ratio

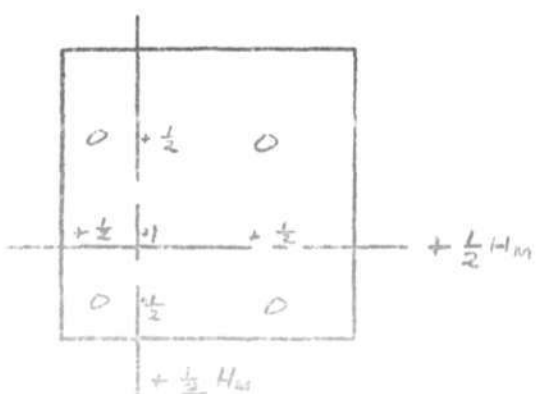
A 2-dimensional system can be arranged to give a 3:1 selecting ratio. The currents to be applied in the two coordinates are as follows when H_M is the drive required to switch:

X coordinate selected	apply	$+2/3\ H_M$
" " unselected	"	0
Y coordinate selected	"	$+1/3\ H_M$
" " unselected	"	$-1/3\ H_M$



X	Y	H_X	H_Y	H
0	0	0	$-\frac{1}{3}$	$-\frac{1}{3}$
0	1	0	$+\frac{1}{3}$	$+\frac{1}{3}$
1	0	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$
1	1	$+\frac{2}{3}$	$+\frac{1}{3}$	+1

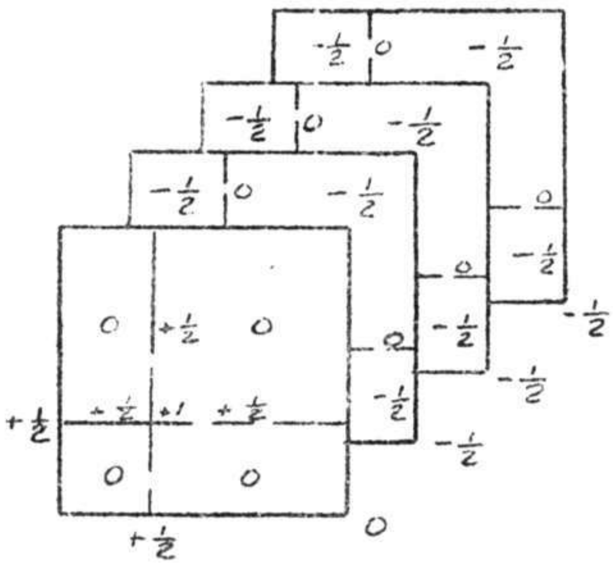
Comparing this system with the present one described in R-187:



X	Y	H_X	H_Y	H
0	0	0	0	0
0	1	0	$+\frac{1}{2}$	$+\frac{1}{2}$
1	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$
1	1	$+\frac{1}{2}$	$+\frac{1}{2}$	+1

3-Dimensional 2:1

The virtue of the last mentioned system appears when another coordinate is added:



X	Y	Z	H_X	H_Y	H_Z	H
0	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
0	0	1	0	0	0	0
0	1	0	0	$+\frac{1}{2}$	$-\frac{1}{2}$	0
1	0	0	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0
0	1	1	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$
1	0	1	$+\frac{1}{2}$	0	0	$+\frac{1}{2}$
1	1	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
1	1	1	$+\frac{1}{2}$	$+\frac{1}{2}$	0	+1

One way of looking at this system is to say that selection is made in all Z planes and then the unwanted planes are overridden or inhibited by a negative H.

There is no equivalent inhibition scheme for the 3:1 system. This lack is a serious restriction since the minimum usable switching system for a parallel computer is 3-dimensional. The absolute minimum is 2-dimensional, one dimension along the digits in a register, the other along the registers. For large numbers of registers the register selection, which is one-dimensional, becomes prohibitive. Note that digit column selection is necessary to allow arbitrarily writing 0's or 1's in each column. The present 3-dimensional system is satisfactory from this point of view since it allows selecting any combination of cores along the Z axis and not just one.

A 3-dimensional system allows 2-dimensional selection of the register number thus reducing the number of drivers to a reasonable level for moderate storage capacities. For very large storage capacities it may be desirable to go to 4- or more-dimensional systems. This possibility lies well in the future.

N-Dimensional Switching

We will consider the problem of selecting a single element from an n-dimensional array of such elements. The selection will be made in the following manner:

1. The selection will be made by n independent linear selections, one in each coordinate.

2. Each linear selection will be of an n-1 dimensional-array.

3. Each element will be at the intersection of n selecting leads, one for each coordinate.

4. The particular selecting arrangement that results in maximizing the ratio of selecting to non-selecting switching signals will be defined as a "best-possible" n-dimensional switching system.

(These restrictions hold for what may be termed "non-redundant" selection systems. Some systems with "redundant" selection are described in the next section.)

Let the selecting amplitude (H_M in Papian's terminology) be taken here as unity drive, and let p be the largest non-selecting amplitude at any core. Now consider a selected core, and then unselect it in one coordinate only; according to restriction (1), above, the other coordinates remain unaffected. Since unselecting must remove a part of the selecting amplitude at least equal to 1-p, unselecting in n coordinates will remove at least n(1-p). As stated, the remaining amplitude of 1-n(1-p) must not exceed p in amplitude; It must not therefore be less than -p since negative disturbance is as bad as positive disturbance. Then:

$$1 - n(1 - p) \geqq -p$$

$$1-n + (n+1)p \geqq 0$$

$$(n+1)p \geqq n-1$$

$$p \geqq \frac{n-1}{n+1}$$

$$p_{Min} = \frac{n-1}{n+1}$$

$$\left(\frac{1}{p}\right)_{Max} = \frac{n+1}{n-1} = R_{Max} = \text{Maximum Selecting Ratio}$$

A tabulation of R_{Max} and p_{Min} vs n follows:

n	$R_{Max} = \frac{n+1}{n-1}$	$p_{Min} = \frac{n-1}{n+1}$
2	3	1/3
3	2	1/2
4	5/3	3/5
5	3/2	2/3
6	7/5	5/7
etc.		

The present system has a p of 1/2 and is "best-possible" 3-dimensional but not "best-possible" 2-dimensional. The 3:1 system described above is "best-possible" 2-dimensional.

A 4-dimensional system according to the above criterion would be, for example:

Coordinates

			X_1	X_2	X_3	X_4
	H	=	+.4	+.2	$\pm$.2	$\pm$.2
or	H	=	+.4	+.4	-.4	$\pm$.2

These two systems are equivalent; of the two, the latter is preferable since the driving equipment is simplified.

In general for an n-dimensional system the coordinate values are:

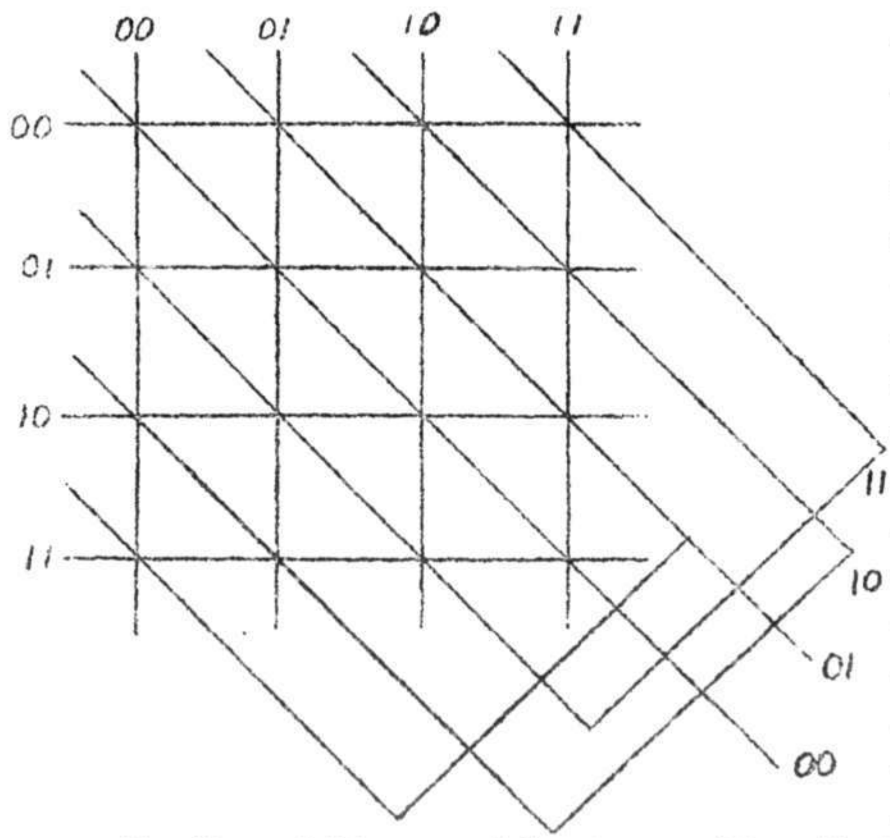
$$\begin{array}{lcccccc} & & X_1 & & X_2 & X_3 \cdots\cdots & X_n \\ H = & + & \frac{2}{n+1} & \pm & \frac{1}{n+1} & \pm\frac{1}{n+1}\cdots\cdots & \pm\frac{1}{n+1} \\ \text{or}\quad H = & + & \frac{2}{n+1} & + & \frac{2}{n+1} & -\frac{2}{n+1}\cdots\cdots & -\frac{2}{n+1}\ (n\ \text{odd}) \\ & & & & & & \pm\frac{1}{n+1}\ (n\ \text{even}) \end{array}$$

Redundant Selection - 2-dimensional

If the restrictions mentioned at the head of the previous section are disregarded, it is possible to devise selection systems that will give selection ratios higher than 3:1. If a ratio of M is desired, a system is needed in which the selected element lies at the intersection of M lines or planes or other configurations which must not otherwise intersect. Since the intersection of just two of these define the element, the

the other N-2 figures are redundant. Moreover, it is in general possible to apply plus or minus voltages to the figures and thus obtain better than an N to 1 selection ratio.

Consider first a 2-dimensional array. Customarily an element is selected at the intersection of one horizontal and one vertical, not-otherwise-intersecting lines. A third group of not-otherwise-intersecting lines are the diagonals.



H	V	Diagonal
00	00	00
00	01	11
00	10	10
00	11	01
01	00	01
01	01	00
01	10	11
01	11	10
10	00	10
10	01	01
10	10	00
10	11	11
11	00	11
11	01	10
11	10	01
11	11	00

The diagonal line cannot be chosen arbitrarily but is a function of the horizontal and vertical lines already chosen. See the table.

The diagonal column may be easily derived from the horizontal and vertical columns by (in this case) subtracting the vertical from the horizontal modulo 4. A physical procedure for this derivation would be:

1. Set the diagonal decoder by the horizontal address digits.

2. Add the complement of the vertical address.

3. Add 1 (corrects for 9's complement).

Since all selected lines are non-intersecting except at one element, a selecting amplitude of 1/3 may be used giving an amplitude at the selected element of 1 and non-selecting amplitudes of at most 1/3.

A better system is possible. All unselected elements in the horizontal and vertical lines must be intersected by the non-chosen diagonal lines (they do not intersect with the chosen diagonal and the diagonal lines pass through all elements). Therefore, a negative signal on the non-chosen diagonals will reduce the non-selecting amplitudes. As a result:

X	coordinate	chosen	+ 2/5
"	"	non-chosen	0
Y	"	chosen	+ 2/5
"	"	non-chosen	0
Diagonal		chosen	+ 1/5
"		non-chosen	- 1/5

The largest non-selecting amplitude is 1/5 resulting in a selecting ratio of 5:1.

There are other 2-dimensional redundant systems. Through the selected element may be drawn a large number of lines of different slopes (the number depends on the size of the array) all of which will pass through 1/n th of the elements, but not all of which are non-intersecting with pre-chosen groups of selecting lines. As an example -- for an array with even n, lines with slope of 3 (up 3 rows for each column) will intersect with X, Y, and diagonal lines only at the mutually selected element. Rules can be worked out for choosing such lines. The resulting selecting ratios are 2m-1:1 where m is the number of groups of lines.

Redundant line selection is also possible in 3 or more dimension arrays but a selection among an n-1 dimensional array of lines is necessary in each group.

In 3 dimensions, the groups could be the 3 coordinates plus the 4 major diagonals. The selected element will be at the intersection of 7 lines. By a method analogous to that described above a selecting ratio of 13:1 may be achieved. The necessary selecting equipment is very complicated and inefficient.

Redundant Selection - 3-Dimensional

Redundant selection is not necessarily restricted to groups of lines. Groups of n-1 dimensional figures may be used in an n-dimensional system.

In 3 dimensions a fourth plane skewed with respect to the major 3 may be used. This plane should intersect with more than one other plane only at the selected element. A plane intersecting the other 3 at 45° fulfills the requirements.

X	coordinate	chosen	+ 1/3
"	"	non-chosen	0
Y	"	chosen	+ 1/3
"	"	non-chosen	0
Z	"	chosen	+ 1/3
"	"	non-chosen	0
Diagonal		chosen	00
"		non-chosen	- 1/3

Applied Signals

Selected element	1
Intersection of any 2 planes	+ 1/3
All planes except at intersections	0
All other elements	- 1/3

Although this is a 3-dimensional system it has the disadvantage that only a single element can be selected and not an arbitrary group of elements along one dimension.

Redundant selecting 3-dimensional systems using more than 4 planes can be devised. The methods can be extended to any number of dimensions.

The Driving Problem

Any reference to the form of storage under consideration has two parts:

1. A "read" or "write minus", the two being equivalent.

2. A "write plus" in selected columns.

Since the read is destructive a rewrite plus in the columns that readout plus is necessary.

Since writing minus requires signals of opposite polarity on all planes from those required when writing plus, write minus must be carried out at a different time than write plus. This difference can most conveniently be obtained by writing minus or clearing all columns prior to the write plus. This write minus is equivalent to reading. It would be possible to write minus only in the columns that are to end up minus but there seems little advantage to such complication.

3-Dimensional Driving

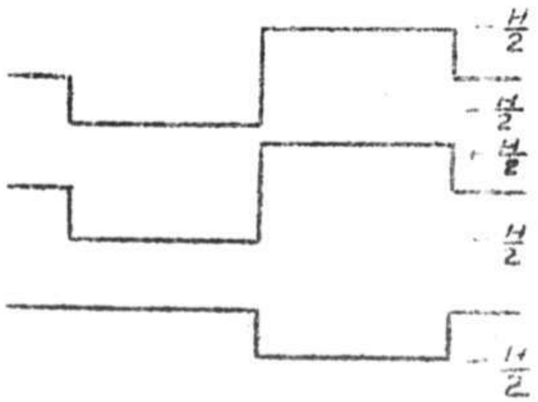
The chosen drivers in the X and Y dimensions always first write minus and then write plus without exception. The drivers in the Z or digit dimension, which are inhibiting drivers, never drive plus (inhibit minus) since all columns are always written negative. Selected Z dimension drivers drive negative driving the write plus of the cycle to inhibit the columns that are not being written plus.

Dimension | Waveform

X

Y

Z in 0 columns only



These waveforms can be obtained with single tubes. The driving sections need not even be push-pull.

As a first approximation a normally "ON" tube may be used with an LC circuit in the plate. An incoming negative gate long enough to allow one complete sine wave on the plate is then applied to the grid. Clipping would be needed to square up the waveform. It may prove desirable to use double-ended drivers to hold constant pulse currents.

A driver of the X, Y kind will be called an s-type driver for sequence-type.

A driver of the Z kind will be called an O-type driver for one-shot type.

A driver which must put out both plus and minus signals but not in fixed order will be called an n-type driver for non-sequenced type. Such a driver would have to be double-ended and is probably more complicated than an s-type driver.

A best-possible 3-dimensional storage of n^2 registers each d digits long requires

2n s-type drivers	nd cores/driver
d O-type drivers8	n^2 " "

2-Dimensional Driving

Consider now a storage made out of best possible 2-dimensional arrays. One such array will be needed for each digit column.

The chosen X-coordinate drivers first drive negative to write minus but drive positive only in the digit columns to be written plus. It seems reasonable, however, that such a driver should be no more complicated than an s-type, the complication appearing in the control circuits. The

Y-coordinate drivers must put out opposite polarities on selected and unselected lines and are therefore of n-type. If a complete set of drivers is provided on each column we need:

nd s-type drivers n cores/driver

nd n-type drivers n " "

If an s-type driver requires 3 tubes

" O-type " " 2 tubes

" r-type " " 4 tubes

and we consider a storage of $32^2 = 1024$ registers of 16 digits each then -

The 3-dimensional array requires 64 x 3 = 192

+ 16 x 2 = 32

224 tubes

The 2-dimensional array requires 3584 tubes

This is a substantial price to pay for the improved selection ratio. No consideration has been given to the fact that the 3-dimensional drivers drive more cores and therefore must be larger than the 2-dimensional drivers. This size difference partially compensates for the different complexity of the two systems.

Fortunately it is not necessary to go to complete separation of digit columns. For example, X-selection can be made in each column, the Y-selections in all columns at once. This arrangement requires

nd s-type drivers n cores/driver

n n-type drivers nd " "

For the hypothetical storage we now need 1669 tubes.

It is possible to omit the n-type drivers completely by biasing the entire 3-dimensional array with a single 1/3 H s-type driver, plus when writing minus, and minus when writing plus. Both x and y drivers can then be s-type also. The same number of x and y drivers as before are required.

2-Dimensional Redundant

This system requires 3 signals in each column, x, y, and diagonal. Two of these can be s-type with an amplitude of 2/5 H, the other is n-type with an amplitude of 1/5 H. A biasing driver such as mentioned above can be used with another set of s-type drivers instead of the n-types. In either event only the ± 1/5 can be common to all columns. The requirements are:

2nd	s-type or	2nd	s-type	n cores/driver
n	n-type	n	s-type	nd " "
		1	s-type	n^2d " "

For the hypothetical storage we now need 3200 tubes.

3-Dimensional Redundant

Any two planes can be common to all columns, the others must be separate. There will be a 3-dimensional array in each column. Therefore we require:

$2\,n^{2/3}$	s-type	$n^{4/3}d$ cores/driver
$2\,n^{2/3}d$	n-type	$n^{4/3}$ " "

The array should be cubical, i.e., 512, 4096, 32768, etc. registers. It may well be that this type of system will be important for very large amounts of storage where 4-dimensions arrays become desirable but where the 5:3 switching ratio of the true 4-dimensional system may be unworkable.

Considering a storage of 16^3 = 4096 registers of 16 digits each, we require:

32	s-type	= 32 x 3	= 96 tubes	4096 cores/driver
512	n-type	= 512 x 4	= 2048 tubes	256 cores/driver
			2144 tubes total	

The 2-dimensional best-possible requires 4288 tubes for this size storage or twice as many.

The 3-dimensional best-possible requires 416 tubes or about one-fifth as many.

Conclusions

It appears possible, at a substantial cost in increased complexity of the associated circuitry, to effectively improve the operating characteristics of any core material by improving the selection ratio. Some preliminary tests made by W. N. Papian show that the response time of a steel core may be approximately halved by using a 3:1 ratio instead of 2:1 and may be halved again by going to 5:1. Some of the recent steel cores are almost fast enough for use in Whirlwind at 2:1. The decision as to whether to use one of the more complicated systems must wait until more information on core characteristics and driver design become available.

Signed Robert R. Everett
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