RECOMP II USERS' PROGRAM NO. 1071

PROGRAM TITLE: AIRBORNE TELLUROMETER - TRILATERATION METHOD

PROGRAM CLASSIFICATION: General

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PURPOSE: To determine the position of an unknown point with reference to the positions of two known points by making a continuous record of slant ranges to the three stations from an aircraft flying along an intermediate path. Calculate the grid distances of this intermediate path using it as the base in the determination of the grid coordinates of the unknown station.

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PROBLEM WRITE UP

1. Task Number: 59-058 Recomp II Computer

2. Date Rec'd: 10 August 1959

3. Submitted by: Mr. Mancini, Air & Grnd Techniques Section, Suveyng & Geod Br.

4. Programmed by: L. A. Gambino

5. Description of Problem: To determine the position of an unknown point with reference to the positions of two known points by making a continuous record of slant ranges to the three stations from an aircraft flying along an intermediate path. Calculate the grid distances of this intermediate path using it as the base in the determination of the grid coordinates of the unknown station.

6. Mathematical Analysis: The trignometric formulae used in the position computations were revised to include the concept from analytic geometry of the directed distance from a line to a point thereby making the formulae applicable to all cases.

7. Numerical Analysis: Newton-Raphson iteration process was utilized to obtain approximate latitudes to be used in the scale factor computations. The coordinates of the unknown can be obtained to ± 0.04 meters.

8. Block Diagram: Attached

9. Program Listing: Attached

10. Operational Notes:

a) Input and output via the electric typewriter. After reading in the tape, the location counter will be set at location 0001.0. Press the "Start 1" button on the right side of the console. The computer will now wait for input data.

1. Type A or B. Type A if the unknown station is above the known base line or type, F if it is below the base line.

2. XXXXXXXX.XXX N_W (Nothing of the west point on the bases.

3. XXXXXX.XXX E_X Easting of the west point on the base.

4. XXXXXXX.XXX (N_E) Northing of the last point on the base.

5. XXXXXX.XXX (E_E) Easting of the east point on the base.

6. XXXX.XXX (H_W) Elevation (in meters) of the west point on the base

7. XXXX.XXX (H_E) Elevation (in meters) of the root point on the base.

8. XXXX.XXX (h_V) Elevation of the unknown station.

9. XX.XXX (TDB_W °F) Dry bulb temperature at west end of base.

10. XX.XXX (TWB_W °F) Wet bulb temperature at west end of base.

11. XXXX.XXX (ALT_W ft.) Atimeter reading (m feet) at west end of base.

12. XX.XXX (E_W) Eccentricity at west end of base.

13. XX.XXX (TDB_E °F) Dry bulb temperature at east end of base.

14. XX.XXX (TWB_E °F) Wet bulb temperature at east end of base.

15. XXXX.XXX (ALT_E ft.) Altimeter reading in feet at east end of base.

16. XX.XXX (E_E) Eccentricity at east end of base.

17. XX.XX (TDB_V °F) Dry bulb at unknown point.

18. XX.XX (TWB_V °F) Wet bulb at unknown point.

19. XX.XX (ALT_V ft) Atlimeter reading in feet at unknown point.

20. XX.XXX (E_V) Eccentricity at unknown point.

21. XX.XXX (TDB°F) Dry bulb at west end of air base.

22. XX.XX (TWB°F) West bulb at west end of air base.

23. XXXX.XX (ALT) Altimeter, in feet at west end of air base.

24. XXXX.XX (h) Altitude of aircraft in meters, at west end of air base.

25. XX.XX (TDB°F) Dry bulb at east end of air base.

26. XX.XX (TWB°F) Wet bulb at east end of air base.

27. XXXX.XX (ALT) Altimeter, in feet, at east end of air base.

28. XXX.XX (h) Altitude of aircraft, in meters, at east end of air base.

29. XXXXX.XXX (W-Air W) Distance, in meters, from West base point to west end of air base.

30. XXXXX.XXX (E- Air W) Distance from E_ast base point to west end of air base.

31. XXXXX.XXX (V - Air W) Distance from unknown to west end of air base.

32. XXXXX.XXX (W - Air E) Distance from West base point to East end of air base.

33. XXXXX.XXX (E - Air E) Distance from East base point to east end of air base.

34. XXXXX.XXX (V - Air E) Distance from unknown point to east end of air base;

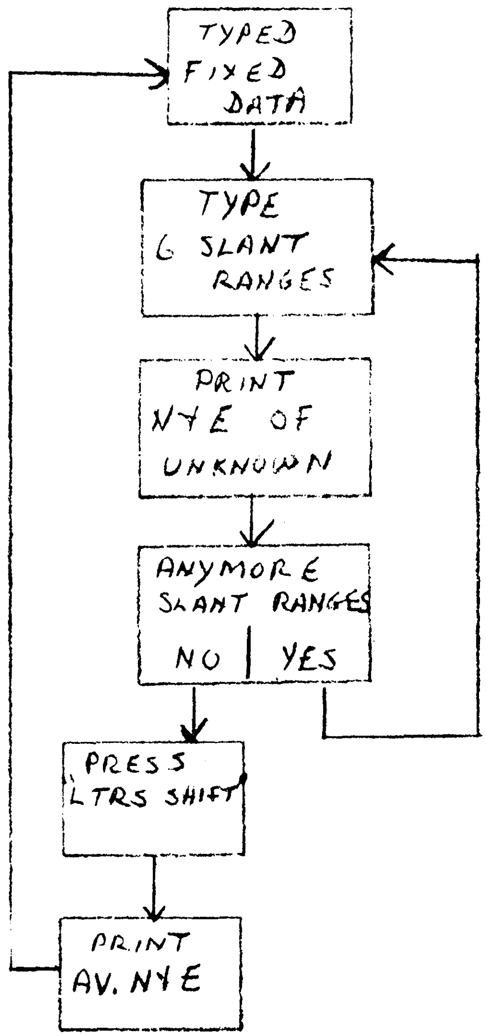
Upon returning the carriage after entering this last distance, the computer goes into the compute mode. After approximately 5 minutes, the position of the unknown will be typed out and the computer will wait for more distances to be entered. If there are no more distances, press the "Letters Shift" on the typewriter and the average of all the computed positions made on the unknown will be typed out. The computer will then set the location counter to 0001.0 and wait for a new problem.

10b. 1. 1608 words or 3216 commands are utilized by this program.

2. Press the carriage return after typing in each piece of data. If an error is made while typing in a number, press the "J" key on the typewriter, press the start button on the console and then retype the number. If an error is detected after entering a number, it is advisable to read the tape back into the computer so that modified commands and stepping constants will be reset to their original values.

59-058

AIRBORNE TELLUROMETER CONTINUOUS TRILATERATION METHOD



U. S. ARMY ENGINEER
GEODESY, INTELLIGENCE AND MAPPING RESEARCH AND DEVELOPMENT AGENCY
SURVEYING AND GEODESY BRANCH

1 February 1961

CALCULATION OF POSITION FROM DISTANCES MEASURED WITH AIRBORNE TELLUROMETER

CONTINUOUS TRILATERATION METHOD

1. General

Electronic distance measuring equipment, which has been given the distinguishing name ' Aerodist ', was developed in the Union of South Africa, for the U. S. Army Engineer Corps (GIMRADA), for long range extension of horizontal control. Essentially, the purpose of the equipment is to determine the position of an unknown point with reference to the positions of two known points, In the diagram below, A and B represent the two fixed ground stations with forward and back azimuths and C is the unknown point whose position is required.

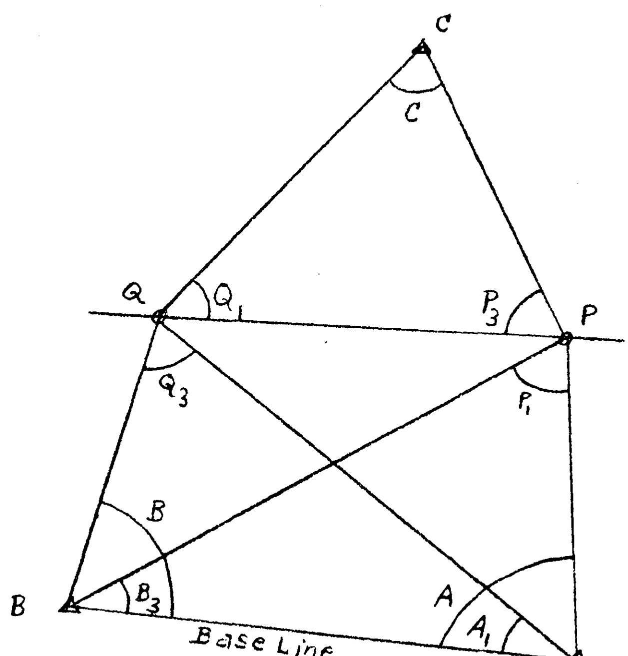


Figure 1

A continuous record of slant ranges to the three stations A, B and C is made in an aircraft flying along an intermediate path, such as P Q. The aircraft is flown at as low an altiutde as possible, consistent with the maintenance of line of sight conditions. In flat terrain, the altitude would be of the order of 2000 feet for ranges of approximately 50 miles. The height determinations of the aircraft and the unknown ground point are made barometrically.

2. Computational procedure

Of the several possible methods of calculation, that which gives accurate results most directly is the computation on the ellipsoid.

The successive stages of computational procedure are:

a. Selection from the continuous record of field data of at least six sets of simultaneously measured slant ranges. Six is considered a minimum, but provision for twelve should be made in the program. Six of these to air point P and six to point Q. For simple illustration in this problem for computational procedure, however, we shall consider only six; three would correspond to positions P_1, P_2 and P_3 in the neighborhood of point P and three to positions Q_1, Q_2, and Q_3 in the vicinity of point Q. Refer to Figure No. 1 for geometry of the problem. To insure optimum results, it is necessary that the figure and especially the triangle PQC be reasonably well conditioned.

b. Computation of index of refraction using meterological data obtained at each point.

c. Reduction of the slant ranges to ellipsoidal distances.

d. Calculation of azimuths from known ground points A and B to air points P_1, P_2, P_3 and Q_1, Q_2, Q_3 projected to the ellipsoid and from Q_1, Q_2, Q_3 to unknown ground point C.

e. Computation of positions of air points and unknown ground point, which will be performed for each set of data with the final ground position taken as the mean of all C values.

3. Reduction of slant ranges to spheroidal distances.

The following procedure of reduction corrects the radio path 'L' to ellipsoid distance S for each measured line. Figure 2 contains the relationship of the variously computed distances to L. All lengths are expressed in meters. h_1 is the elevation of ground point and h_2 the elevation of aircraft (also in meters).

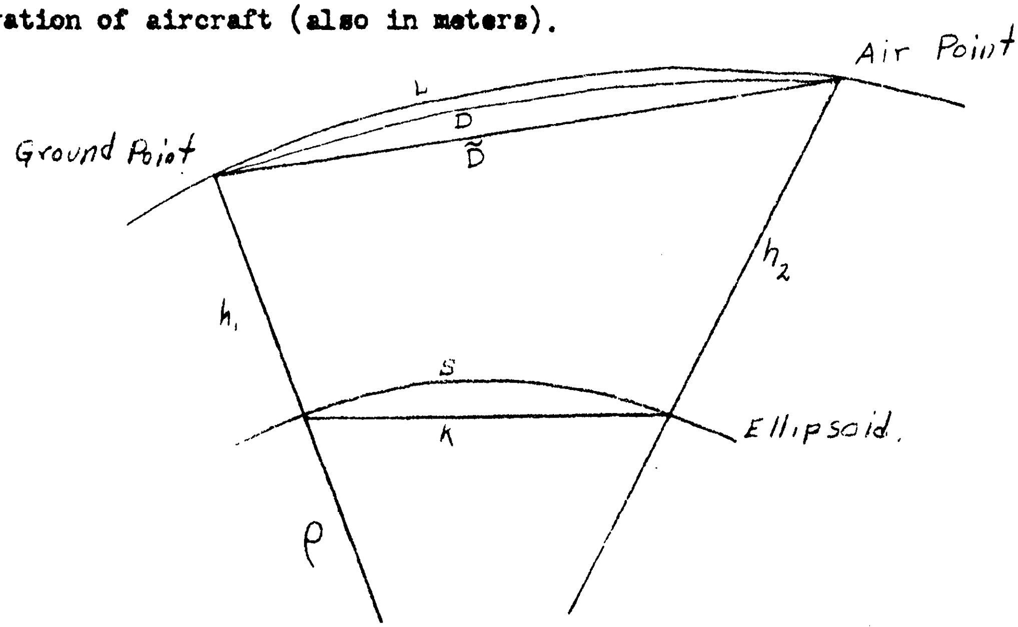


Figure 2.

a. Reduction of L to D.

The reduction of each radio path L to slant range D corrects for index of refraction (η) and is accomplished by application of the method contained in the Airborne Tellurometer Computations, Line Crossing Method (2 Feb 1961). The meteorological data will be measured and treated in the same manner as in the Line Crossing Problem.

b. Reduction of D to $\tilde{D}$.

Slant ranges are next reduced to slant chord distances $\tilde{D}$ for each line AQ_1, AP_1, BQ_1, BP_1, Q_1C_1, P_1C_1, etc., by the formula

$$\tilde{D} = D - D^3/24r^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where $r = 26.0 \times 10^6$ meters (radius of radio wave).

c. Reduction of $\tilde{D}$ to K.

1. In performing the reduction of $\tilde{D}$ to ellipsoidal chord distances K, approximate azimuths of all measured lines and geographic coordinates of points P_1, P_2, Q_1, Q_2, etc., must first be determined. By giving the various distances their respective azimuths prior to further reduction makes possible a refinement of the distance since in many cases these measured lengths are likely to be in excess of 100 miles. Formulas contained on page 1 of Problem No. 8, Trilateration Adjustment (3 April 1958) of the Engineers Problems can be used for computing internal angles of the triangles of the quadrilateral. The azimuths of lines AQ_1, AQ_2, BP_1, BP_2, etc., need only be computed. Azimuths may be added according to the convention set forth in the Line Crossing Method.

2. Next, the approximate geographic coordinates of the air points P_1, P_2, Q_1, Q_2, etc., are required. These coordinates can be determined by employing Helmert's Position computation which is a part of this problem and utilizes as input data initial coordinates of A(orB) and azimuth to the airpoint Q(orP).

3. Following these preliminary computations, the reduction of $\tilde{D}$ to K is relatively simple. By application of the following formula utilizing azimuth and position data thus determined, the ellipsoidal chord distance K is obtained.

$$K = \sqrt{\frac{\tilde{D}^2 - (h_2 - h_1)^2}{\left(1 + \frac{h_1}{\rho'_\alpha}\right)\left(1 + \frac{h_2}{\rho'_\alpha}\right)}} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

where $\tilde{D}$, h_1, and h_2 are known,

and

$$\rho'_\alpha = \rho\nu/(\nu\cos^2\alpha + \rho\sin^2\alpha) = \text{radius of curvature.}$$

also $\rho = a\,(1-e^2)/(1-e^2\sin^2\phi_m)^{3/2}$

$\nu = a/(1-e^2\sin^2\phi_m)^{1/2}$

further $a = 6378206$ m = semi-major axis of ellipsoid.

ϕ_m = latitude of starting point of line.

α = azimuth from A to Q_1, B to P_1, etc., from north (previously computed).

d. Reduction of K to S.

Finally, to obtain ellipsoidal arc distance S, we use the relation

$$S = K + \frac{K^3}{24\rho_\alpha'^2} + \frac{3K^5}{640\rho_\alpha'^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where K and ρ_α' are defined above.

e. Adjustment of triangles.

Each triangle ABP_1, ABP_2, ABQ_1, ABQ_2, etc., is now ready for final adjustment. By utilising the formulae

$$\cos B_3 = \frac{\cos \overline{PA} - \cos \overline{BP} \cos \overline{BA}}{\sin \overline{BP} \sin \overline{BA}} \quad \ldots\ldots(4)$$

$$\cos A = \frac{\cos \overline{BP} - \cos \overline{BA} \cos \overline{PA}}{\sin \overline{BA} \sin \overline{PA}} \quad \ldots\ldots(5)$$

$$\cos P_1 = \frac{\cos \overline{AB} - \cos \overline{AP} \cos \overline{BP}}{\sin \overline{AP} \sin \overline{BP}} \quad \ldots\ldots(6)$$

.....................................,

compute the angles of the two triangles the sum of each should = $180^\circ + \epsilon$, where ϵ = spherical excess to be computed according to eq. (14) in Line Crossing Method, with ϕ_m = mean of vertices of each triangle.

Since this condition rarely occurs, there will normally be an error of closure E of the triangle. The error of closure is to be eliminated according to the procedure in Line Crossing Method, page 7.

f. Final azimuth.

Final azimuth of lines BP_1, BP_2, AQ_1, AQ_2, etc. can be determined by adding (or subtracting) the angles A_1, A_2, B_1, B_2, etc., to the azimuth of line AB (known) according to the convention on page 7 of the Line Crossing Method.

g. Computation of line PQ.

1. The final geographic coordinates of points P_1Q_1, P_2, Q_2, etc., are to be computed from the final azimuths above and the ellipsoidal lengths 'S' previously computed. Helmert's Position Computation is to be used.

To compute the length P_1Q_1, P_2Q_2, etc., a copy of Sodano's inverse formula (with accompanying example) is also furnished and will be used for the computation. This formula supplies equations for computation of forward and back azimuth, also.

h. Computation of unknown point C.

1. From the computed lengths and asimuths of line PQ, the approximate azimuths of lines Q_1C_1, QC_2, P_1C_1, P_2C_2, etc., are computed along with coordinates for C_1, C_2, etc. Internal angles of the triangle $P_1Q_1C_1$, $P_2Q_2C_2$, etc., are adjusted in the same manner as above for the triangles of the quadrilateral, except this time lines P_1Q_1, P_2Q_2, etc., are considered as the base line for each set of data instead of AB as before.

2. Final azimuths Q_1C_1, Q_2C_2, etc., are determined as above in f.

3. The geographic coordinates of point C_1, C_2, etc., from Q_1, Q_2, etc., only are computed and finally, the latitudes and longitudes of the various values of C are meaned to determine the most probable value.

i. Synopsis.

Since the problem is set fourth in some detail, a quick review of computational procedure for single value of C is felt desirable. This review is supplied as an aid to the programmer in hopes of simplifying the problem.

From the given data of stations A and B, coordinates of air points P_1 and Q_1 (reduced to the ellipsoid) are computed over lines BP_1 and AQ_1 as a function of azimuth and distance. An inverse of line P_1 Q_1 is performed, and using this line as a base with azimuth Q_1 C_1 and the related distance, geographic coordinates for C_1 are computed. This procedure is carried out for each set of measurements resulting in a unique value for C. The C's are then meaned as described above to yield the most probable value for the unknown point.

j. Program flexibility.

The program will be developed for optional input and required output of the following basic geodetic data. This problem operates on the ellipsoid (geodetic); and should input data of the starting points be on the UTM plain or in mils, then it will be required that these data be converted to geodetic quantities via subroutines (integral part of the program).

1. Input.

   a. Geodetic

      (1). Geographic coordinates (° ′ ″)

      (2). Azimuths (° ′ ″)

   b. UTM.

      (1). Coordinates (meters) and zone, converted by subroutine to geographic coordinates.

      (2). Azimuth (° ′ ″ on mils), converted by subroutine to geodetic azimuths.

2. Output.

   a. Geodetic

      (1). Geographic coordinates (° ′ ″)

      (2). Azimuth (° ′ ″)

   b. UTM.

      (1). Coordinates (meters) and zone.

      (2). Azimuth (° ′ ″ and mils).

k. Utilize formulae of Engineer problem no. 12 (UTM to geographics) to convert UTM coordinate input to geographics. From other existing formulae convert UTM sexagesimal or mil azimuth to geodetic.

l. In output utilize Engineer survey problem 11 formulae for Geographic to UTM coordinates. From convergence formulae, convert geodetic azimuth to UTM sexagesimal and mil azimuths.

m. Entire program will contain as integral part all subroutines.

n. Program will be designed to work only north of the equator and west of Greenwich.

o. Program shall be developed so that following entry of Baseline data, new Field data can be entered for computing other C's.

Note: Refer to Format as guidance.