Analysis of Filter Coefficient Precision on LMS Algorithm Performance for G.165/G.168 Echo Cancellation

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1. Introduction

In a digital implementation of the LMS algorithm, the adjustable filter coefficients and the signal levels are quantized to within a least significant digit. By doing so, we introduce an additional error, quantization error. The effect of quantization error in the filter coefficient is of particular interest because it determines the choice of the processor to be used in the implementation to meet the application requirements. In following we will focus our discussion on the effects of finite filter coefficient precision on the LMS algorithm performance in line echo cancellation using TMS320C6201 digital processor. The study is conducted through theoretical analysis and computer simulation. The error in signal quantization is considered negligible for the simplicity of discussion.

2. Theoretical Analysis

In this analysis, a transversal FIR filter is used to predict the echo from the history of the far-end signal, and the echo residue is calculated as:

$$e(n) = d(n) - \frac{1}{A}\sum_{k=0}^{M-1} H_k(n)x(n-k) \qquad (1)$$

where *e(n)* is the value of residue at time *n*, *d(n)* is the value of echo at time *n*, $H_k(n)$ is the *k*th filter coefficient at time *n*, and *x(n-k)* is the value of the far-end signal at time *n-k*. *M* is the length of the filter, which is determined by the echo tail length. *A* is the normalizing factor such that the filter output will have the same precision as *d(n)*.

The filter is updated by the LMS algorithm as:

$$H_k(n+1) = H_k(n) + \mu e(n)x(n-k) \qquad (2)$$

where $\mu \geq 0$ is the adaptation step size.

For the leaky LMS algorithm, the filter is updated by:

$$H_k(n+1) = \beta H_k(n) + \mu e(n)x(n-k) \qquad (3)$$

where $0 \leq \beta \leq 1$ is the leaky factor, which is introduced to acquire more control of the filter response.

There are primarily two effects due to the quantization error in filter coefficients. The first effect is the accuracy degradation in the calculation of echo prediction due to the quantization error in filter coefficients. Assume the following model for the filter coefficients H_k:

$$H_k = H_{0k} + e_k$$

where e_k is the error term.

The filter output is calculated as

$$y = \frac{1}{A}\sum_{k=0}^{M-1} H_k x_k = \frac{1}{A}\sum_{k=0}^{M-1} H_{0k} x_k + \frac{1}{A}\sum_{k=0}^{M-1} e_k x_k$$

In order to evaluate the error term $\Delta = \frac{1}{A}\sum_{k=0}^{M-1} e_k x_k$, e_k and x_k are assumed to be independent random variables with zero mean value such that its variance is given by

$$\sigma_\Delta^2 = \frac{1}{A^2} M\sigma_e^2\sigma_x^2 \qquad \text{and} \qquad \sigma_\Delta = \frac{1}{A}\sqrt{M}\sigma_e\sigma_x$$

For filter coefficients with 16 bit precision (using TMS320C6201), the final result of the accumulation of products are normally shifted right by 16 bit (A=65536) to form a filter output with the same precision of the echo signal *d(n)*. Since G.165/G.168 specification requires that echo residue is about 30 dB below the far end input signal level, the quality of cancellation is guaranteed as long as

$$\sigma_\Delta << \frac{\sigma_x}{31.6}$$

As an example, we assume that e_k is uniformly distributed in {-0.5, 0.5}, and x_k is treated as a full strength PCM signal uniformly distributed in {-4096,4096}. For a channel with 48ms echo tail (*M=384*), we have

$$\sigma_\Delta = \frac{1}{65536}\sqrt{384\frac{1}{12}\frac{4096^2}{3}} = 0.204$$

Since the 99% confidence interval of the error term $3\sigma_\Delta = 0.612$ is less than 1, even the LSB of the filter output would not be affected. The error introduced by 16 bit filter coefficient quantization is negligible in computing the filter output.

For a channel with 64ms echo tail, *M=512*.

$$\sigma_\Delta = \frac{1}{65536}\sqrt{512\frac{1}{12}\frac{4096^2}{3}} = 0.250$$

Since the 99% confidence interval of the error term $3\sigma_{\Delta} = 0.75$ is less than 1, The error introduced by 16 bit filter coefficient quantization is negligible in computing the filter output for the same reason.

The second effect of the filter coefficient quantization is the early "digital cutoff" of the algorithm. In a quantized LMS filter, the algorithm will stop making any further adjustment to the filter coefficients when the correction term $\mu e(n)x(n-k)$ in equation 2 is less in magnitude than half of the filter coefficients quantization interval. The performance would be degraded because the adaptation is terminated by quantization effect.

The LMS algorithm will continue to adapt if

$$|\mu e(n)x(n-k| \geq 2^{-B-1}$$

where B is the number of bits used to represent the filter coefficients. This condition can be approximated by replacing the magnitude by its RMS value. That is,

$$\mu \cdot \sigma_e \cdot \sigma_x \geq 2^{-B-1}$$

It was shown in [1] that for a M-tap LMS filter in a severely distorted channel, the maximum permissible value of the adaptation step size is given as

$$\mu_{\max} = \frac{1}{M\sigma_x^2}$$

It is common to set the step size as half of the maximum permissible step size [1] so that,

$$\frac{1}{2M\,\sigma_x/\sigma_e} \geq 2^{-B-1}$$

That is,

$$B \geq 3.322(\log_{10} M + \log_{10}(\sigma_x/\sigma_e))$$

The above expression indicates that the minimum number bits required for the filter coefficients are proportional to the logarithm of the filter length and signal-to-noise ration requirements.

ITU G.165 and G.168 specifications require to achieve about 30 dB cancellation ($\log_{10}(\sigma_x/\sigma_e) \geq 1.5$) when the filter converges. For a channel with 48 ms echo tail, *M=384*. The minimum value of *B* is solved to be 13.6. For a channel with 64 ms echo tail, *M=512*. The minimum value of B is 14 bits. Filter coefficients with 16 bit precision seems to be adequate for these applications.

3. Simulation Results

Several cases of simulation of the leaky LMS algorithm using the code from reference [2] were conducted for 32 ms and 48 ms echo tail. A 500 ms simulation interval is chosen to meet the G.165/G.168 specification. Figure 1 and Figure 2 illustrate the simulation results for 32 ms and 48 ms echo tail at various far-end signal levels. The far-end signal level changes from 0 dB (0dB=4096) to -24 dB in a 6 dB step. It can be seen that the filter for 32 ms echo tail converges after about 200 ms while the filter for 48 ms echo tail converges after about 300 ms. It is because that the adaptation step size has to be smaller for the filter with longer length in order for the adaptive filter to converge. It was shown in [1] that the maximum permissible step size is inversely proportional to the filter length.

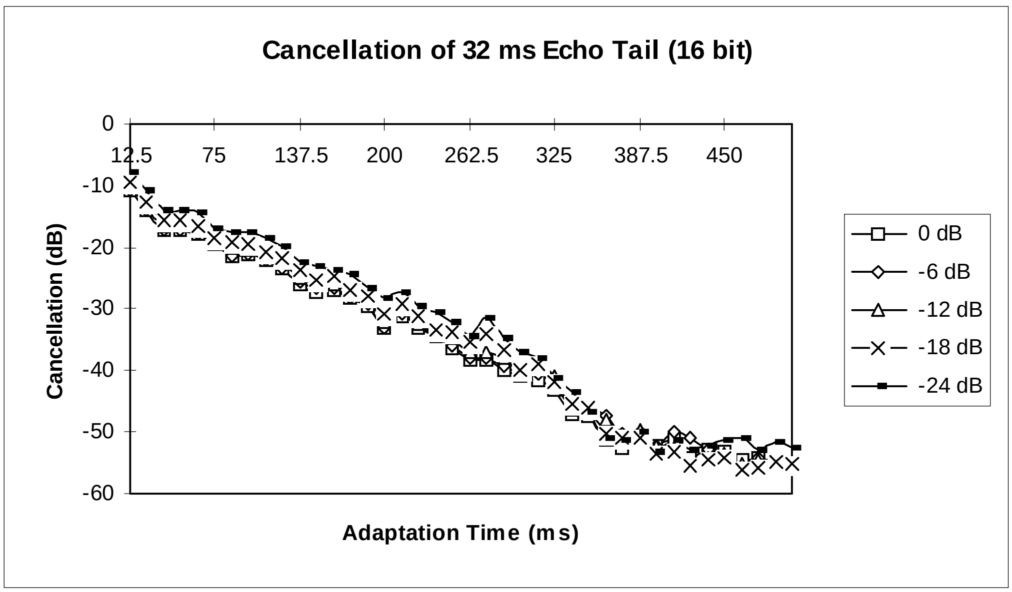

Figure 1: Simulation with 32 ms echo tail and 16 bit filter coefficients.

The simulation results of 16 bit implementation are also compared to those of 32 bit implementation in Figure 3, Figure 4, and Figure 5 for echo tails of 32 ms, 48 ms, and 64 ms respectively. The far-end input signal level in the simulation is chosen at 0 dB. The adaptation step size is the same for both implementations. In Figure 6 and Figure 7, the input signal level is set at -30 dB in order to illustrate the filter characteristics at low input signal level. No significant discrepancy on convergence rate is observed. This also verified the theory that the number of bits used to represent the filter coefficients does not affect the convergence rate, but only affect the final convergence level. It has been shown that the convergence rate of an LMS algorithm only depends on three factors: the step size, the number of filter taps, and the nature of the input signal [3]. The lower convergence rate at low input signal level could be the contribution of the quantization error in echo residue *e(n)* when *e(n)* is very small, which is not related to filter precision.

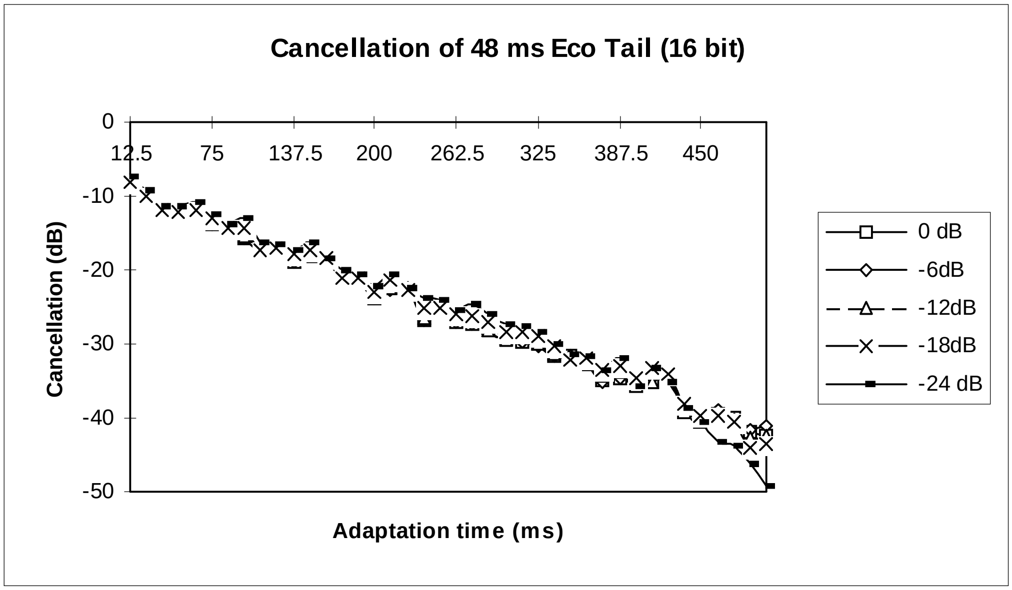

Figure 2: Simulation with 48 ms echo tail and 16 bit filter coefficients

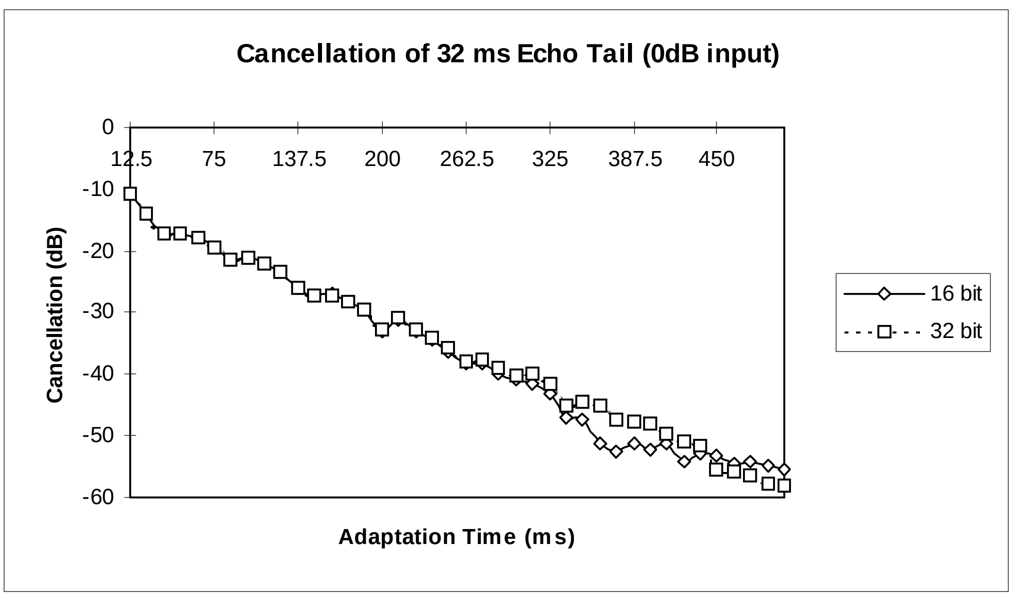

Figure 3: Comparison between 16 and 32 bit filter coefficients: 32 ms echo tail.

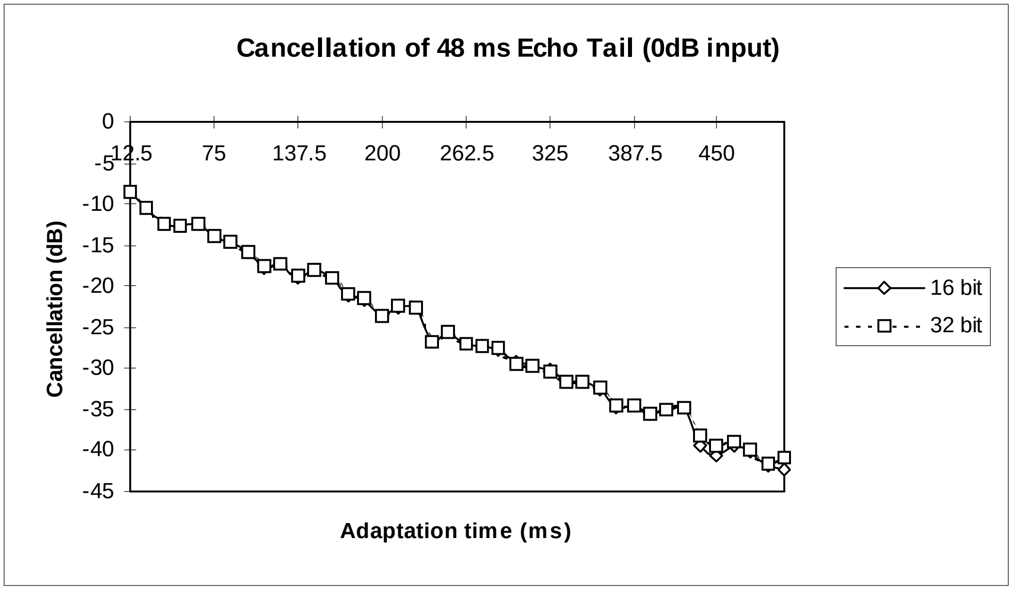

Figure 4: Comparison between 16 and 32 bit filter coefficients: 48 ms echo tail.

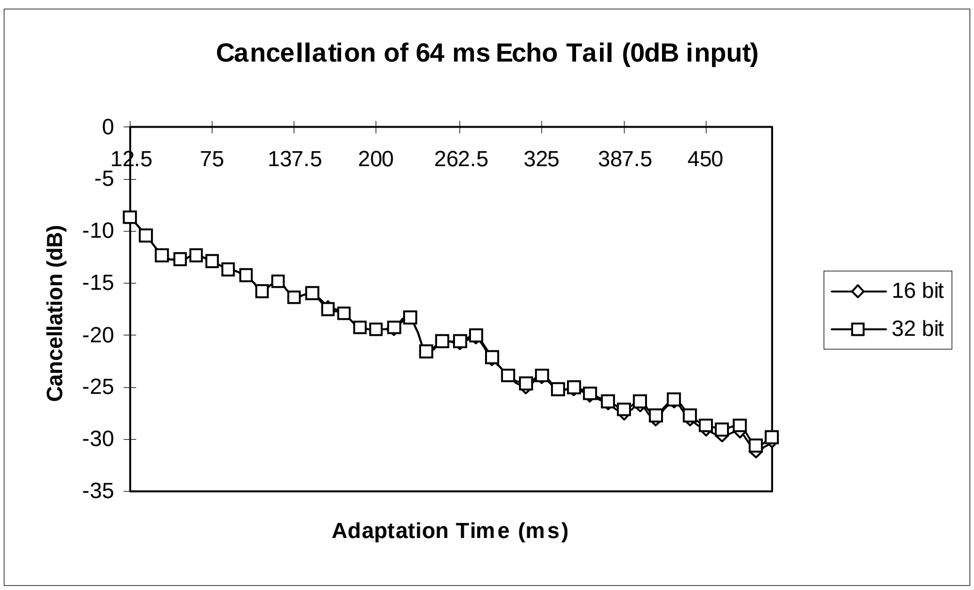

Figure 5: Comparison between 16 and 32 bit filter coefficients: 64 ms echo tail.

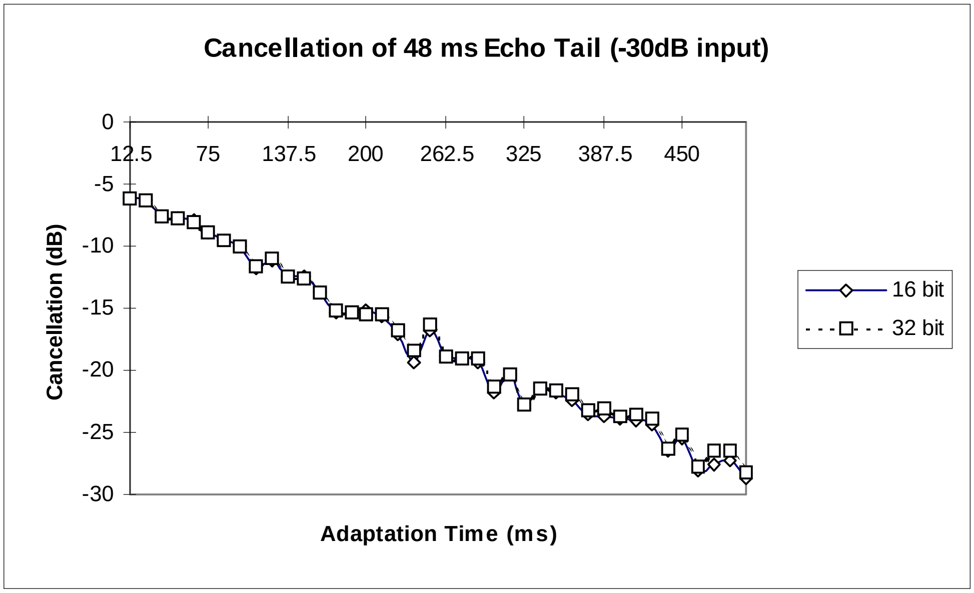

Figure 6: Comparison between 16 bit and 32 bit filter coefficients: 48 echo tail.

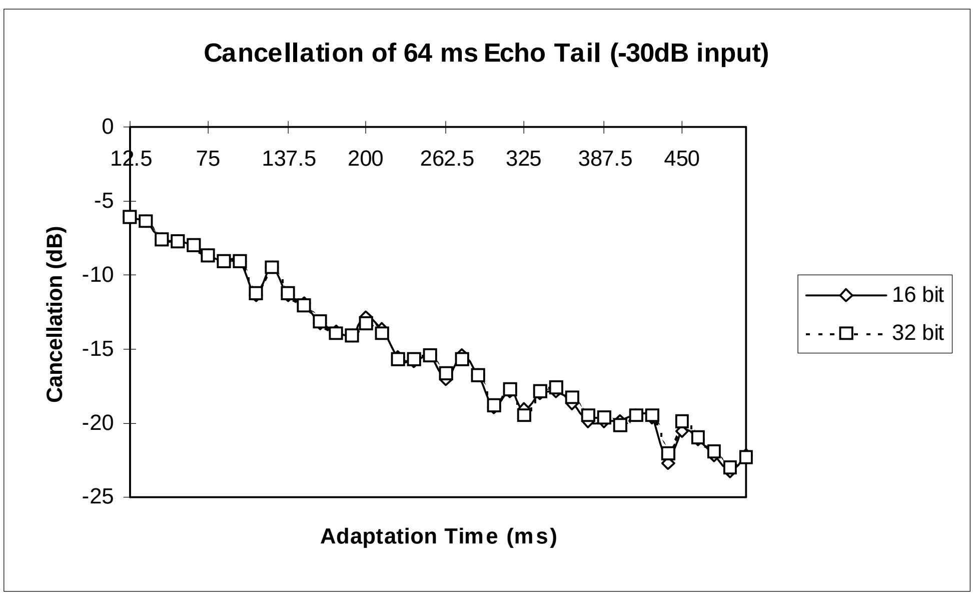

Figure 7: Comparison between 16 and 32 bit filter coefficients: 64 ms echo tail.

4. Conclusion

An analysis of the effect of the LMS filter coefficients quantization error has been presented in this memo. The results of theoretical analysis and computer simulation have shown that the performance of the echo canceler based on 16 bit filter coefficient will meet G.165/G.168 requirements.

5. References

[1] Gitlin and Weinstein, “On the required tap-weight precision for digitally implemented, adaptive, mean-squared equalizers”, Bell System Technical Journal, Vol. 58, No. 2, 1979

[2] “Implementation of echo control for G165/DECT on Texas Instruments TMS320C62xx processors”, Texas Instruments application report, August 1997

[3] Simon Haykin, “Adaptive filter theory”, Prentice-Hall, 1986